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PHYSICAL REVIEW.

THE INFLUENCE OF ELECTRIFICATION UPON THE SURFACE TENSION OF WATER AND MERCURY.[1]

By Ernest Merritt and Samuel J. Barnett.

THE investigations of Exner and Tuma,[2] Warburg,[3] G. Meyer,[4] Luggin,[5] S. Meyer,[6] Nernst,[7] Behn,[8] S. W. J. Smith,[9] and many others have established beyond reasonable doubt that the electrocapillary phenomena occuring at the interface between a liquid metal and an electrolyte, in a capillary electrometer, for example, are due almost entirely to true alterations of surface tension resulting from the chemical changes of electrolysis. There are many other electrocapillary phenomena, however, which are undoubtedly caused largely, if not wholly, by the electrostatic forces that are developed by the presence of a charge upon the liquid surface. Thus when electrified, a soap bubble expands, a globule of mercury becomes more flattened, the wave pattern of a liquid jet issuing from an elliptical aperture becomes more elongated,[10] etc., etc. In such cases the effect of electrification is usually similar to that which would be produced by a *diminution* of the surface tension. This

[1] A paper read before the American Physical Society, December 28, 1899.
[2] Wien. Ber. 97 IIa, p. 923, 1888.
[3] Wied. Ann. 38, p. 321, 1889; ibid., 41, p. 1, 1890.
[4] Wied. Ann 45, p. 508, 1892; ibid., 53, p. 845, 1894; et al.
[5] Wien Ber. 102 IIa, p. 913, 1893.
[6] Wien. Ber. 105 IIa, p. 139, 1896.
[7] Zeitschrift für Elektrochemie, 4, p. 29, 1897.
[8] Wied. Ann. 61, p. 1897.
[9] Proc. R. S. L. 64, p. 253, 1899.
[10] Phil. Mag. Sept. 1882, p. 186.

fact is probably due to the universal employment of convex surfaces in such experiments; in the case of a concave charged surface a consideration of the electrostatic forces would lead us to expect an apparent increase in the strength of the surface film.

To determine whether, in addition to this electrostatic effect, there exists a true effect of electrification upon the surface tension of a liquid, investigations must obviously be made by methods which make the calculation of the purely electrostatic effect feasible, so that it may be eliminated from the total effect. Several earlier investigations bearing upon the existence and magnitude of a true change in surface tension produced by electrification, although quite inconclusive so far as such an effect is concerned, were reviewed by one of us in a recent paper.[1] This paper also described a new and more elaborate investigation of the same problem, and showed that the effect in question, if it exists at all, is much smaller than previously supposed. As the theory of the method used had not been fully developed at the time of publication, no more definite conclusion could then be drawn. To complete this theory, and by means of it to give the observations their final discussion, is the object of the present communication.

For a detailed account of the experiments, reference must be made to the paper already mentioned. It is sufficient to state here that the tension of the liquid surfaces, uncharged and charged to known electrical densities, was investigated by the method of capillary ripples, in a form capable of giving the surface tension of unelectrified water with an accidental error not greater than 0.05%.[2] When the liquids were electrically charged, either positively or negatively, the velocity of the capillary waves was found to be less than when the surface was unelectrified. In the case of water this observed change would correspond to a diminution in surface tension of about 4% for a surface density of 1.75 electrostratic units. For mercury the apparent change was not so great, being slightly less than 1% for a surface density of 2.33. It is to be observed, in accordance with what has been said above, that the results cited do not necessarily prove the existence of a *real* change in surface ten-

[1] Barnett, PHYSICAL REVIEW, 6, p. 257, 1898.
[2] Barnett l. c., p. 263.

sion; for the electrostatic forces acting upon a charged surface would cause a diminution in the velocity of capillary ripples even if the surface tension remained unaltered. In order to determine whether a real change in surface tension results from electrification, we must take account of the electrostatic forces in deriving the relation between surface tension and velocity.

To determine the velocity of capillary ripples we have the general equation[1]

$$\frac{\delta p}{\rho} = -gh - \frac{d\varphi}{dt} \tag{1}$$

where ρ is the density of the liquid, h the displacement of the point considered above its equilibrium position, δp the resulting increment of pressure at the surface, φ the velocity potential, and g the acceleration of gravity.

If δp is due to capillarity only, we have

$$\delta p = -T\frac{\partial^2 h}{\partial x^2}$$

T being the surface tension; and equation (1) becomes

$$\frac{T}{\rho}\frac{\partial^2 h}{\partial x^2} = gh + \frac{d\varphi}{dt}. \tag{2}$$

For the case of plane sine waves such as we have supposed, this gives for the velocity v

$$v^2 = \left(\frac{g}{k} + T\frac{k}{\rho}\right)\tanh kl$$

l being the depth of the liquid, λ the wave-length, and $k = \frac{2\pi}{\lambda}$.[2]

In the experiments with which we are concerned the value of l was such that $\tanh kl$ differed from unity by less than one part in 2000, and we may put

$$v^2 = \frac{g}{k} + T\frac{k}{\rho}$$

with an error less than the experimental error.[3]

[1] Rayleigh, Treatise on Sound, Vol. II, § 353, Second edition.
[2] Rayleigh, Treatise on Sound, Vol. II, § 353, Second edition.
[3] Barnett *l. c.*, p., 263.

If the wave frequency is n we have $v = n\lambda$, and the final expression for the surface tension becomes

$$T = \frac{\rho\lambda^2}{2\pi}\left(\lambda n_2 - \frac{g}{2\pi}\right). \tag{3}$$

When the surface of the liquid is charged, however, a pressure $-\frac{2\pi\sigma^2}{K}$ is developed in addition to that due to capillarity, σ being the electric surface density and K the dielectric constant of the medium above the waves. In order to obtain the whole increment of pressure, δp, resulting from the displacement h, it is therefore necessary to add to $-T\frac{\partial^2 h}{\partial x^2}$ a term $-\frac{2\pi}{K}(\sigma^2 - \sigma_0^2)$ where σ_0 is the surface density of the charge on the undistorted surface. In the place of equation (2) we have therefore, in this case,

$$\frac{T}{\rho}\frac{\partial^2 h}{\partial x^2} + \frac{2\pi}{K\rho}(\sigma^2 - \sigma_0^2) = gh + \frac{d\varphi}{dt}. \tag{4}$$

The surface density σ at any point of the wave surface is a function of h whose form remains to be determined.

In the experiments already mentioned the surface of the liquid was made one coating of an air condenser, the other coating being formed of tinfoil supported on glass at a height of about 35 mm. above the surface. This arrangement was adopted in order to obtain a more intense charge upon the liquid surface, and to make the electrical conditions definite, so that all the quantities involved should be capable of exact determination. To find the distribution of the charge over the wave surface under the conditions of the experiment, we may proceed as follows:

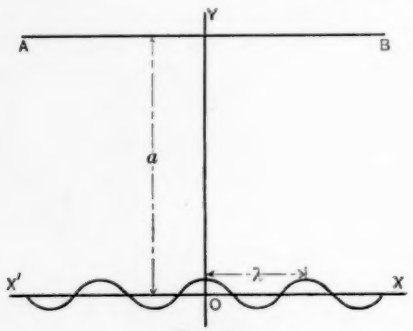


Fig. 1.

Let the axis of X be taken in the original level surface of the liquid, while the axis of Y is normal to the surface. Consider a

train of plane waves of amplitude b travelling in the direction of X. As the waves proceed a rearrangement of the charge occurs; but this takes place so rapidly that the electrical conditions are always the same as though the ripples were stationary. The surface being at a potential V_0, while the plate AB is grounded, we have to determine the value of σ for all points along XX'.

For the region between the plates AB and the wave surface XX', the electrical potential V is a function of x and y subject to the condition

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0. \tag{5}$$

It is clear that V, as a function of x, is periodic in the distance λ. With the origin as shown in the figure we may therefore write

$$V = A_0 + A_1 \cos kx + A_2 \cos 2kx + \cdots. \tag{6}$$

Since V must have the same value for $-x$ as for $+x$, none but cosine terms will appear in the expansion. The coefficients A_0, A_1, etc., are functions of y, but are independent of x.

Upon substitution in (5) we obtain

$$\frac{\partial^2 A_0}{\partial y^2} + \left(\frac{\partial^2 A_1}{\partial y^2} - k^2 A_1\right) \cos kx + \left(\frac{\partial^2 A_2}{\partial y^2} - 4k^2 A_2\right) \cos 2kx + \cdots = 0.$$

Since this equation holds for all values of x the coefficients must vanish separately. Therefore,

$$\frac{\partial^2 A_0}{\partial y^2} = 0, \quad \frac{\partial^2 A_1}{\partial y^2} - k^2 A_1 = 0, \text{ etc.}$$

The solution of these equations leads to the following values for the coefficients in (6):

$$A_0 = M_0 y + N_0$$
$$A_1 = M_1 \varepsilon^{ky} + N_1 \varepsilon^{-ky}$$
$$A_2 = M_2 \varepsilon^{2ky} + N_2 \varepsilon^{-2ky}$$
$$\text{etc.}$$

The constants of integration are to be so chosen as to satisfy the boundary conditions, viz.: (1) For $y = a$, $V = 0$ for all values of x: (2) for $y = b \cos kx$, $V = V_0$.

The first condition requires that A_0, A_1, etc., should separately vanish for $y = a$. Therefore

$$M_0 = -\frac{1}{a} N_0$$
$$M_1 = -N_1 \varepsilon^{-2ka}$$
$$M_2 = -N_2 \varepsilon^{-4ka}$$
$$\text{etc.}$$

For the coefficient A_1 we therefore have

$$A_1 = N_1 (\varepsilon^{-ky} - \varepsilon^{-2ka+ky}).$$

Since the expression for V is to be used only in order to find the force at the surface of the liquid, and from this the value of σ, we are chiefly concerned with its value for points immediately above the surface. For such points y is small when compared either with a or with λ. The exponent of the second term of A_1 for points in the immediate neighborhood of the surface is therefore practically equal to $-2ka$. In the experiments upon water the value of a (distance between the liquid surface and the plate above) was about 4 cm., while λ was always less than 0.5 cm. In the experiments upon mercury, the value of a was about 3.4 cm., and that of λ less than 0.37 cm. In all cases the quantity $2ka$ is therefore greater than 100, and the second term is the expression for A_1 is entirely negligible. This is also true of the corresponding terms in A_2, A_3, etc. The expression for V therefore takes the form

$$V = -N_0 \frac{y}{a} + N_0 + N_1 \varepsilon^{-2ky} \cos kx + N_2 \varepsilon^{-2ky} \cos 2kx + \cdots. \quad (7)$$

For points on the wave surface, where $y = b \cos kx$, the potential is V_0. Therefore for points on the surface

$$V_0 = N_0 - N_0 \frac{b}{a} \cos kx + N_1 \varepsilon^{-ky} \cos kx + N_2 \varepsilon^{-2ky} \cos 2kx + \cdots.$$

Remembering that the amplitude, b, of the ripples is extremely small, and that the y in the above expression can in no case exceed b, it is clear that the exponentials are practically equal to unity; and we have as a close approximation,

$$0 = N_0 - V_0 + \left(N_1 - \frac{b}{a} N_0\right) \cos kx + N_2 \cos 2kx + \cdots.$$

Since this relation holds for all values of x, the coefficients N_2 N_3, etc., vanish, and

$$N_0 = V_0, \quad N_1 = \frac{b}{a} V_0.$$

The final approximate expression for V is therefore

$$V = V_0 \left(1 - \frac{y}{a} + \frac{b}{a} \varepsilon^{-ky} \cos kx\right) \tag{8}$$

and that for Y, the vertical component of the force at the surface, is

$$Y = -\frac{\partial V}{\partial y} = \frac{V_0}{a}(1 + kb \cos kx).$$

For waves of the extremely small amplitude used the horizontal component of the force is neglible in comparison with Y. Therefore,

$$\sigma = \frac{KV_0}{4\pi a}(1 + kb \cos kx).$$

Remembering that $\frac{KV_0}{4\pi a} = \sigma_0$, that $kb \cos kx$ is small, and that $b \cos kx = y = h$, we have, finally,

$$\sigma^2 - \sigma_0^2 = 2kh\sigma_0^2 = \frac{4\pi\sigma_0^2}{\lambda} h$$

and equation (4) becomes

$$\frac{T}{\rho} \frac{\partial^2 h}{\partial x^2} = \left(g - \frac{8\pi^2\sigma_0^2}{K\lambda\rho}\right)h + \frac{d\varphi}{dt}. \tag{9}$$

Upon comparing this with equation (2) it appears that the effect of electrifying the surface is to alter the velocity of the waves in the same sense and to the same extent as would a diminution of the acceleration of gravity by $\frac{8\pi^2\sigma_0^2}{K\lambda\rho}$.

The corrected expression for the surface tension [see (3)] is, therefore,

$$T = \frac{\rho\lambda^2}{2\pi}\left(\lambda n^2 - \frac{g}{2\pi}\right) + \frac{2\lambda\sigma_0^2}{K}. \tag{10}$$

The *apparent* surface tension *i. e.*, the value of T obtained from (3) [or from (10) without taking into account the final term, which

gives the correction for electrostatic influences] is thus always less than the true surface tension by the amount $\frac{2\lambda\sigma_0^2}{K}$.

TABLE I.

Water.

V	λ	σ_0	$\frac{2\lambda\sigma_0^2}{74.3}$	T Apparent.	T Corrected.
0	0.496	0.00	0.00%	100.00%	100.00%
9000	.496	0.58	0.45	99.61	.06
12000	.495	0.77	0.79	99.29	.08
15000	.495	0.97	1.25	98.98	.23
18000	.494	1.17	1.82	98.37	.19
21000	.493	1.36	2.46	97.69	.15
24000	.492	1.56	3.22	96.94	.16
27000	.491	1.74	3.00	96.13	.13

TABLE II.

Mercury.

V	λ	σ_0	$\frac{2\lambda\sigma_0^2}{403.4}$	T Apparent.	T Corrected.
0	0.369	0.00	0.00%	100.00%	100.00%
9000	"	0.70	.09	99.92	.01
12000	"	0.93	.16	.88	.04
15000	"	1.17	.25	.83	.08
18000	"	1.39	.35	.75	.10
21000	"	1.63	.49	.61	.10
24000	"	1.87	.64	.44	.08
27000	"	2.09	.80	.25	.05
30000	"	2.33	.99	.08	.07

The experimental values of this apparent tension of water and mercury have been given for a number of different values of σ_0 in Tables IV. and V. of the paper referred to above, expressed in percentage of the values for $\sigma_0 = 0$.[1] By the use of equation (10) we are now able to apply the correction due to electrostatic influences. The results are given in Tables I. and II., expressed as before in percentage of the maximum values of T. These maximum values were 74.3 $\frac{\text{dynes}}{\text{cms}}$ for water and 403.4 $\frac{\text{dynes}}{\text{cms}}$ for mercury, respectively,

[1] Barnett, l. c., pp. 281–282.

which were obtained by making the following substitutions in (10):

$$K = 1, \quad g = 981, \quad n = 64.4, \quad \sigma_0 = 0$$

also, for water

$$\lambda = 0.496, \quad \rho = 0.998$$

and for mercury

$$\lambda = 0.369, \quad \rho = 13.54.$$

The final column of Table I. gives for the corrected surface tension of an electrified water surface values ranging from 0.06% to 0.23% higher than that of the uncharged surface, the average value being about $\frac{1}{7}$% higher; while the corresponding column of Table II. gives for the tension of an electrified mercury surface values from 0.01% to 0.10% higher than that for no electrification—the mean being about $\frac{1}{15}$% higher. The fact, however, that the very small variation follows no law in either case, being often less for high than for low electric surface densities, would indicate that it possesses no significance, and is due wholly to experimental errors. To make this evident, we shall examine the accidental error in measuring the "apparent" surface tension of water; what is said of the experiments upon water being true, with little alteration, for those upon mercury. As was shown in the paper already often quoted, a given percentage error in measuring λ produces in the computed value of T, under the conditions of the experiments, a percentage error about three times as great. More accurately, $\frac{\Delta T}{T} = 2.9 \frac{\Delta \lambda}{\lambda}$. An examination of figure 6 (*l. c.*) shows that in measuring λ the average departure from the mean (given by the curves) in each of the two measurements necessary was about $\frac{1}{20}$%. The average accidental error in T is therefore about $\frac{1}{4}$%; so that the small differences in the corrected value of the surface tension are all within experimental errors, and we may safely conclude that no effect of electrification upon the surface tension has been observed.

The complete investigation, therefore, while not excluding the possibility of discovering an effect of electrification upon surface tension by the use of improved methods and still more intense charges, itself gives no support to the view that such an effect exists.

AN OBJECTIVE REPRESENTATION OF THE HYSTERESIS OF IRON AND STEEL.[1]

KNUT ÅNGSTRÖM.

THE very happy thought of using cathode rays for the study of alternating currents and of alternating magnetic fields appears to have had its origin with Mr. Braun.[2] The method employed is obvious from Fig. 1, which shows a tube of the kind furnished by Geissler in Bonn, according to Braun's specifications. Cathode rays from the terminal K pass through a small hole in the diaphragm D, and produce a brilliant spot of light upon that portion of the screen S upon which they fall. This screen, which is covered with phosphorescent material, fills the end of the tube. The ap-

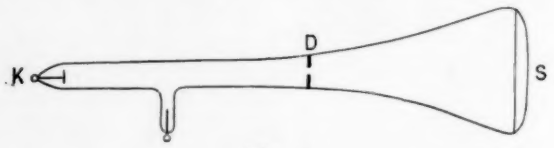


Fig. 1.

proach of a magnetic pole at D deflects the spot of light. If a small magnetizing coil, through which alternating currents flow, be fastened beneath the diaphragm, the spot of light is thrown into a vibratory motion which is capable of analysis by means of the revolving mirror. If two coils, placed perpendicular to each other, with their axes at right angles to the axis of the tube are fastened in the neighborhood of the diaphragm, and each of these coils be supplied with alternating current, the spot of light under the influence of the two forces is given a combined motion analagous to Lissajous's curves.

[1] Translated from the transactions of the Royal Academy of Sciences, Stockholm, 1899, No. 4, by E. L. N.

[2] Braun. Wiedemann's Annalen, 60, p. 552, 1897; Elektrotechnische Zeitschrift 19, p. 204, 1898. The latter article is known to me only through an abstract in Wiedemann's *Beiblätter*, 22, p. 884, 1898.

Braun, in addition to his other experiments, has investigated the velocity of the transmission of magnetization in iron. Since these experiments bear a certain relation to some of those which follow, I take the liberty of describing them in a translation of his own words.[1]

"An iron rod, 2 m. long and 9 mm. in diameter, lies in a horizontal position, perpendicular to the axis of the tube, its end as near as possible to the diaphragm. Along this rod, a small magnetizing coil may be moved. A second coil precisely similar to this, is arranged in a vertical plane, likewise perpendicular to the tube, so that under the simultaneous action of an alternating current in the two spools, the spot of light describes a curve, the form of which is determined by the pole lying nearest to the diaphragm. Both coils are traversed by the same alternating current.

"If the coil is moved along the rod, the shape and position of the ellipse of vibration changes and when the middle of the spool is about 42 cm. from the end of the rod a difference of phase of $\frac{\pi}{2}$ shows itself which is independent of the strength of the current. From the number of oscillations (50) of the alternating current it follows that the velocity of 'propagation' of magnetic excitation is eighty-six $\left(\frac{\text{meters}}{\text{seconds}}\right)$, a value which accords very well with that found by Oberbeck under similar conditions (88.7 meters for a rod 8.7 mm. in thickness, with a vibration frequency of 133)."

As Braun himself remarks, we have to do here with complicated phenomena, and it is a question whether the value of the velocity of transmission thus determined is really that of the magnetic excitation.

While engaged in repeating some of Braun's experiments, I asked myself whether it would not be possible by means of his apparatus to represent objectively the hysteresis curves in iron and steel and to exhibit the same, and after several preliminary attempts I found a simple arrangement which led, in a most beautiful manner, to the result. If a piece of iron or steel is magnetized in a magnetic field, for example in a long magnetizing coil, and if we plot a curve with rectangular coördinates in which the strength of the field H is taken as abscissa and the induced magnetism I as ordinate, we obtain, as

[1] Wiedemann's Annalen, 60, p. 557.

is well known (when H passes through values from H_1 to $-H_1$), a curve which we call the hysteresis curve, and which owes its existence to the fact that induced magnetism in iron does not possess the same value when increasing and when decreasing for equal values of H. This curve and the surface enclosed by the same, which represents a certain amount of energy consumed in the cycle of magnetization, may be determined in a variety of ways and when changes in the magnetizing force take place slowly the process is one which offers no great difficulties. The relations are, however, much more complicated when the intensity of the field changes rapidly.

In the latter case losses of energy occur, as is well known, not only in consequence of the actual phenomena of hysteresis, but also in consequence of the action of Foucault currents. These which are generated in part by the inducing current in the mass of the iron, and in part by the induced magnetism itself, cause a retardation of the magnetic action upon the deeper layers of the substance (electro-magnetic screening). Attempts to separate these various factors from one another and to study them in detail have been made in recent times, among others by Ch. Maurain,[1] and by Wien.[2]

From their work one can form a clear conception of the difficulties with which such investigations are surrounded, and of the advantage of finding a simple method which would allow this relationship to be easily determined, which would make it possible to gain a ready acquaintanceship with the subject, and which, in other ways, would furnish a certain check upon the results. This has been attempted by Ewing,[3] who has constructed an apparatus that records the curves of magnetization automatically. In this apparatus a wire, which is placed between the poles of a permanent magnet, is traversed by a magnetizing current. Another wire carries a constant current and the latter is subjected to the action of the induced magnetism in the iron. The forces that act upon these wires turn a little mirror about two axes situated at right angles one to another. When the iron passes through a complete cycle of magnetization, a beam of light reflected from the mirror records upon a photographic plate

[1] Maurain; Annales de Chimie et de Physique; Series 6, 14, p. 208 (1898).
[2] Wien; Wiedemann's Annalen; 66, p. 859 (1898).
[3] Ewing; The Electrician; May, 1893.

the curve of magnetization. These and similar methods can, in the nature of the case, only be used where the changes in the magnetizing force take place slowly, since, as is the case with all mechanical devices, the inertia of the many parts introduces disturbances whenever the period of vibration is rapid.

First arrangement of my apparatus.—The first arrangement which I used will be easily seen from Fig. 2. The four coils, S, S_1, M, M_1, are fastened around the diaphragm in the manner shown. The dimensions of the coils are as follows:

S, length 2 cm., external diameter 1.6 cm., internal diameter 0.5 cm., diameter of wire 0.3 mm.

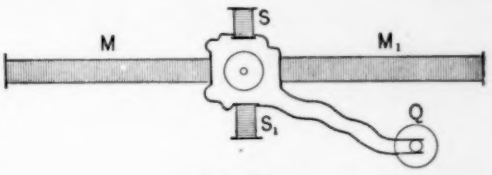


Fig. 2.

M, length 20.5 cm., number of turns, 30 per second.

The coils, S and S_1 were wound upon wood, M and M_1 upon glass tubes. The current from a source Q passes through these four coils in series. It flows through S and S_1 (the indicator coils of Braun), in such a manner that the magnetic action of the same at the opening of the diaphragm is additive; through M and M_1 in such directions that their action at the point is annulled. The horizontal deflection produced by means of the action of the current in S and S_1 is proportional to the strength of the current and also to the magnetizing force. If a rod of iron be introduced into one of the magnetizing coils, the spot of light will be deflected in a vertical direction with a force which is proportional to the induced magnetism, so that under these circumstances the spot of light must pass through a true hysteresis curve.

Second arrangement.—The two indicator coils S and S_1 retain their places, but they are slightly displaced backwards and forwards respectively, with reference to the plane of the diaphragm. The magnetizing coils M and M_1 are placed parallel to each other and at equal distances from the diaphragm as shown in Fig. 3. The action of the current and of the magnetic field is the same as in the first arrangement. When currents of rather high intensity are used, it is easily possible to follow the movement of the spot of light

directly with the eye. If, on the contrary, the cycle of magnetization is gone through with slowly, and especially if it is desired to measure the curve of hysteresis with precision, a photographic record is very advantageous. I set up the photographic camera in such a way that the images of the curves should be taken in half size. An exposure of from ten to twenty seconds suffices for the production of a good negative, which when developed and fixed, can be enlarged to any desired size by projection. The figures in the accompanying plate are reproduced directly from such negatives. The curves shown in the plate are from experiments upon hard steel wires 10 cm. in length and 0.175 cm. in diameter. I. was obtained from a wire containing 0.1 per cent.; II. from a wire containing 0.4 per cent. and III. from a wire containing 0.9 per cent. of carbon. I.–II., I.–III. and II.–III. are "difference curves" between these (I., II. and III.) taken two and two.

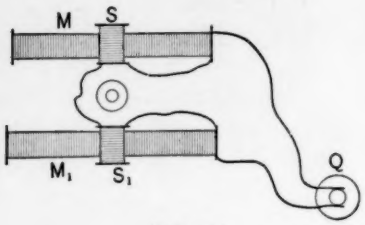


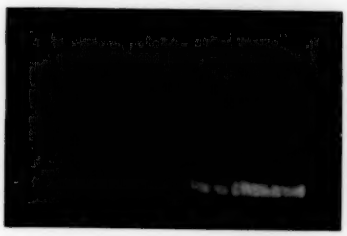
Fig. 3.

For very slow cycles, of magnetization (to produce static curves of hysteresis) I used as a source of current a storage battery, and varied the strength of the current by the introduction of liquid resistance, so as to change the magnetizing force between the desired limits. Alternating current was produced from a small hand-driven Siemen's alternating dynamo (from Ducretet in Paris) with which it was possible to vary the number of alternations between twenty and sixty per second. In this paper only a few examples of these experiments are given. I hope later to return to a more complete description of the results. Figs. 4–8 show drawings from some of the photographic records obtained with the second arrangement of the apparatus. The curves marked *A*, *B*, *C*, *D* refer to the following samples of iron:

(*A*) A rod of iron; length 10 cm., diameter 0.3 cm., amount of carbon 0.2%; tempered. (From Bofors in Sweden.)

(*B*) A rod of iron; length 10 cm., diameter 0.3 cm., amount of carbon 0.8%; hardened. (From Bofors in Sweden.)

(*C*) A bundle of very soft tempered iron wire, length 10 cm., diameter of each wire 0.082 cm.



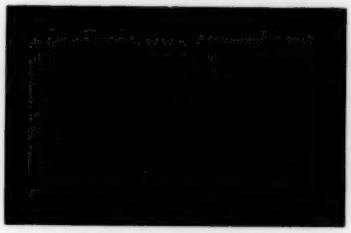
I

I–II

II

I–III

III



II–III

ÅNGSTRÖM: HYSTERESIS OF IRON AND STEEL.

(*D*) A precisely similar bundle of wires, surrounded by a tube of brass. External diameter of the tube 0.48 cm., internal diameter 0.30 cm.

Figure 4 shows the static curve of hysteresis. Figure 5 the corresponding curve for alternating current of about twenty reversals per second. Figure 6 shows the same curve for about sixty rever-

Fig. 4.

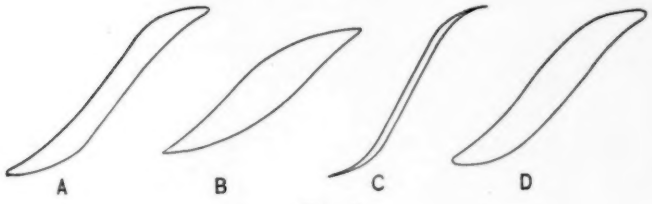


Fig. 5.

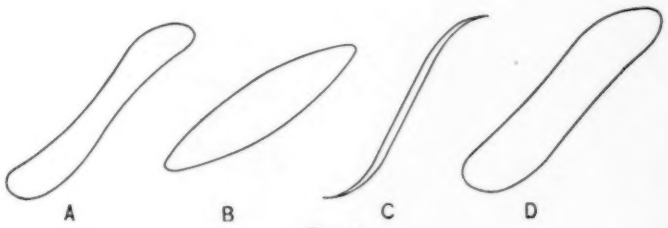


Fig. 6.

sals per second. The maximum strength of the current in these experiments was maintained as nearly as possible constant, with a value of two amperes. In the static hysteresis curves obtained by this method, the influence of hardness is easily recognized. In the case of very soft iron there is naturally no difference between a massive rod and a bundle of wires. In massive rods of soft iron with increasing frequency of alternations, the loops change greatly but in the case of hard iron, the change is insignificant. That these

changes are almost exclusively due to the Foucault currents is evident from the experiments with the bundle of wires with and without the brass tube. The change in the loop with the number of alternations could not be detected in the case of the bundle, without the brass tube, in which case Foucault currents are almost completely excluded, but when the bundle was surrounded by the brass tube, the change in the loop was almost as great as in the case of a soft rod of massive iron.

In general, these results are in good agreement with the experiments of Maurain and of Wien in so far as they are capable of comparison with those investigations. Wien has used alternations up to a frequency of five hundred and twenty, and has studied the changes in the real hysteresis curves. Maurain finds that in the case of bundles of very fine wire, the loss of energy in the cycle of magnetization up to periods of fifty-five alternations per second is not appreciably dependent upon the frequency.[1] Unfortunately, I have not as yet had the means at my disposal to extend these investigations to a very high frequency.

A very interesting modification of the above mentioned experiment can be made by providing the magnetizing coils M and M_1 with iron cores. The curves obtained in this way are difference curves, which depend upon the varied permeability and hysteresis of the specimens, as well as upon the strength and difference of phase which the Foucault currents in the same possess. Figures 7 and 8 afford examples of such difference curves for two different frequencies, obtained by the used of the two rods, A and B, just described. In this case the difference curve for low frequencies is principally due to the difference in hysteresis. At higher frequencies, we have to do, in addition to the above, with unequal changes of phase of the Foucault currents. The curves in figure 8 were

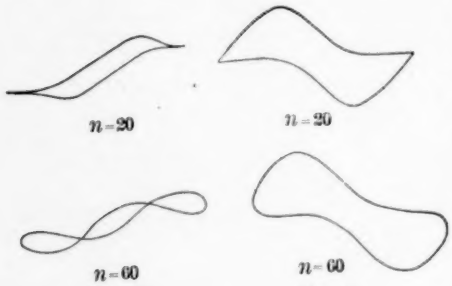


Fig. 7.

Fig. 8.

[1] Maurain, l. c., p. 279.

obtained by means of the bundles of wires, *C* and *D* (with and without the brass rod). In this instance, the hysteresis remains unchanged, and the difference curve is to be ascribed exclusively to the action of the Foucault currents. This last named arrangement of the experiment affords a simple and practical method of testing various samples of iron in respect to their magnetic properties, and likewise to the amount of carbon which they contain.[1] If we place in the magnetizing coils two precisely similar rods of iron of the same character, they neutralize each other and the spot of light moves in a horizontal straight path just as if no iron were placed in these coils. If the rods, however, possess different properties, we obtain more or less well marked difference curves. If, therefore, we have samples of iron, the properties of which are precisely known, we can compare with them directly the property of unknown samples of iron of the same dimensions. The usefulness of this application of the method in electro-technical work, and particularly in the construction of dynamo machines, where hysteresis is of great importance, is obvious.

If, in the first arrangement of the apparatus the rods of iron are brought with their ends very near the opening of the diaphragm and if the magnetizing coils in the second arrangement are fastened directly above and below the rods, and we compare the results obtained in these ways, with one another, it is found, particularly if the iron rods are of great length, that there is a very marked difference between the two curves. The hysteresis curves trend more slowly upward and downward in the first case than in the second. I was for a time at a loss to explain these facts, but I believe that the phenomenon is due to changes in the magnetic distribution within the rods with increasing magnetization. The following experiments confirm this opinion. If we make use only of the two long magnetizing coils, and arrange these as shown in Figure 9, we find when the diaphragm is at the point *a*, symmetrically located with reference to the two spools and when a bundle of iron wires is thrust into one of the tubes, that vertical vibrations take place. If now the diaphragm is pushed along to the point, *b*, *c*, etc., the di-

[1] This comparison is based upon the assumption that the iron in other respects is of similar chemical and physical structure.

rection of vibration tends to turn itself in such a manner as to be always perpendicular to the lines of the magnetic field. At the same time, we notice, however, that the line of vibration as is shown in Figure 9, becomes curved at the ends. If we replace the bundle of wires with an iron rod, we no longer get lines of vibration, but complete loops. With increasing intensity of magnetization, the system of lines of force is displaced, and the whole action is like a shifting of the poles towards the ends of the magnetized rod. If this shifting is not the same for increasing and diminishing magnetization, or in other words, it is connected with a hysteresis effect, we obtain the above described loops. Through this displacement of the poles, we may likewise explain the above difference in the curves obtained with the first and second arrangements of the apparatus.

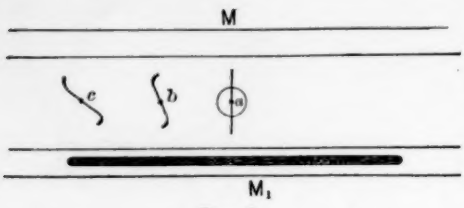


Fig. 9.

When short rods are used, and at a considerable distance from the diaphragm, the difference between the curves obtained is unimportant. The precise measurement of these curves would probably teach us much concerning the permeability and hysteresis of iron. Before entering upon this work in the case of certain varieties of Swedish iron, I purpose, however, to attempt important improvements in the cathode tube. I hope then to be able to work with currents of much higher frequency.

To Mr. Granqvist of the physical laboratory at Upsala, who assisted me in most of these experiments, I desire to express my heartiest thanks.

A DETERMINATION OF THE NATURE OF THE ELECTRIC AND MAGNETIC QUANTITIES AND OF THE DENSITY AND ELASTICITY OF THE ETHER, II.

REGINALD AUBREY FESSENDEN.

Summary.

IT was first shown that if we consider only those electric and magnetic phenomena which can be exactly expressed by equations containing only electric, magnetic and gravitational quantities, with those of length, mass and time, that there are an infinite number of theories possible.

If we suppose that the electric and magnetic quantities are such as we meet with in mechanics the number of possible theories will be much reduced.

We have no right to make this assumption, however, and the reduction cannot be regarded as rigorous, but rather as having a certain degree of probability or as serving as an indication, and hence can only be justified by showing that the results obtained by its means are in accordance with observed facts.

On this basis it was shown that one of two theories is the correct one, *i. e.*, that either $\varkappa$ is a density and μ a compliancy or μ is a density and $\varkappa$ a compliancy. It was then shown that if either coefficient is a compliancy, then, when the corresponding intensity is varied, the corresponding coefficient should vary in the opposite direction.

If neither theory were correct this change should not necessarily take place.

It was found that there is such a relation existing between one of the coefficients and the corresponding intensity, *i. e.*, between μ and H, whilst there is no such relation observable between the other coefficient and intensity.

It was then shown that the rate of change of the coefficient μ with the intensity H was exactly that called for by the theory and a most rigorous experiment showed no deviation from the rate required not within the limits of experimental error.

It was shown that this relation held not only for iron and other strongly magnetic substances, but also for a wide range of substances whose reluctivities varied from 0.0003 to 1000, and was hence probably a universal relation.

Corroborative evidence was then given, showing that the reluctivity varied in different materials, in the same materials under temporary stress and in the same materials under permanent strain, in such a way as to accord with the theory. It was also shown that there was a relation between the coefficients of reluctance and of hysteresis which was also in accordance with the theory.

It was then shown that there was evidence of such a connection between capacity and density as agreed with the theory.

From this last relation the density and elasticity of the ether were determined, to a first approximation.

Conclusion.

Having determined the qualities of the electric and magnetic quantities, we may next proceed to more precisely determine their nature. For convenience we will here reproduce equations 12–15.

$$Q = M/T, \tag{12}$$

$$P = L^2, \tag{13}$$

$$\varkappa = M/L, \tag{14}$$

$$\mu = LT^2/M. \tag{15}$$

The complete determination of these qualities is made as follows:

From equation 3, Maxwell's equation, we have

$$\varkappa^{\frac{1}{2}}\mu^{\frac{1}{2}} = T/L = 1/\text{velocity}.$$

Let us take the X axis as in the direction of the lines of electric flux, the Y axis as in that of the lines of magnetic flux, and the Z axis as perpendicular to both.

The velocity of light is therefore to be measured along the Z axis. We know also that in the expression for volume, *i. e.*, L^3, the three lengths are to be taken in three different directions.

We have therefore

$$M/L_xL_yL_z \times \mu = T^2/L^2_z$$

whence $$\mu = L_xL_yT^2/ML_z.$$

Again, we have

$$B^2/8\pi\mu = \text{work per c. c.} = (P/\text{area})^2/8\pi\mu.$$

This work is done in the Y direction,

hence $$P = L_xL_y \text{ and } B = L_y/L_z.$$

From the equation

$$H\mu = B$$

we have $$H = ML_y/L_yL_xT^2 = M/L_xT^2.$$

To recapitulate,

$$B = L_y/L_z$$
$$H = ML_y/L_yL_xT^2,$$
$$\mu = L_xL_yT^2/ML_z.$$

From this we see that a current flowing in a circuit (this being in the X axis) produces a twisting force around the axis of the wire, H being the intensity of the force, or the force per sq. cm. This force produces a shear, B being the amount of the shear, and the shear at any point depending upon the distance from the wire. The amount of the shear for a given force per sq. cm. depends upon the resistance of the medium to shearing stress, and $1/\mu$ is the rigidity modulus of the medium. Hence $B = H\mu$.

In the presence of iron the virtual rigidity is greatly reduced. The stress H produces a shear B_e in the ether numerically equal, if we take the permeability of the ether as unity, to H. The same stress also produces a virtual shear B_i, ($= 4\pi \times$ intensity of magnetization). The sum of these two shears makes up the total shear B, or $B = B_e + B_i$ (equals numerically $H + 4\pi I$).

The shear B_e increases directly as H. Since the greatest value of H which we can reach is not more than 100,000, which corresponds to a force of only about 4.10^8 dynes per cm. and the rigidity of the ether is, as we have found, approximately 6.10^{20}, we could hardly have expected it to be otherwise, as the greatest shear which we can produce is through but an infinitesimal angle.

The shear B_i (for it is virtually equivalent to a shear), increases as H increases, but reaches a limit, for iron, at about $B_i = 25{,}000$. It does not increase however, directly as H, but as we have seen the two are connected by the formula

$$B_i = H/(a + bH).$$

Under certain circumstances, for example with stressed nickel, as shown in Fig. 4, the constant a is reduced to zero and the intrinsic shear takes its full value with an indefinitely small value of the stress H, any additional increase in B being due entirely to increase of the extrinsic flux.

It will be noted that P/μ is a force in the Z direction.

Taking up next the electric quantities, from the formula

$$D^2/8\pi\varkappa = (Q/\text{area})^2/8\pi = \text{work per c.c.}$$

we get, since the work is done along the X axis,

$$Q = M/T$$

$$\text{and } D = M/L_yL_zT.$$

From the formula

$$QE = \text{work},$$

the work being done along the X axis,

$$E = L_x^2/T$$

$$F = L_x/T.$$

To recapitulate,

$$D = M/L_yL_zT$$

$$F = L_x/T$$

$$\varkappa = M/L_xL_yL_z$$

We may interpret this as meaning that a varying magnetic field produces a flow in the ether in a plane perpendicular to the direction of the magnetic lines, which flow persists, with varying velocity, so long as the amount of the shear is changing. The electric intensity F at any point is the velocity of the flow at that point. The capacity, $\varkappa$, is the density of the flowing medium and the displacement per square cm. D, is the current density.

If we have a condenser with a dielectric part paraffin, of density 0.9 and part vacuum, then since the electrostatic induction is con-

tinuous through the interface, *i. e.*, M/L_yL_zT is constant, the velocities must be different for each side of the interface.

The density of the ether is 0.6, *i. e.*, there is that much inertia per c.c. in the vacuum. In the space occupied by the paraffin the total amount of inertia is $0.9 + 0.6 = 1.5$.

Therefore, since the flow per sq. cm. is constant, and the densities vary in the ratio 0.6 : 1.5, the velocities, *i. e.*, the electric intensities, must vary in the inverse ratio. Hence the slope of electric potential changes at the interface in the ratio 1.5 : 0.6.

But the energy is given by the formula $mv^2/2$. Therefore in the ether the energy per c.c. is 5/3 of that in the paraffin.

The velocity in the ether corresponding to any voltage may be calculated, on the assumption that all the ether flows. We have

$$\varkappa F^2/8\pi = \text{energy per c.c.}$$

and $\varkappa$ is 0.6. If F is one electrostatic unit and $\varkappa$ is taken as 1 for the ether, we have,

$$v^2 \times 0.3 = 1$$

and

$$v = 1.8 \text{ cm. per sec.} = F/\sqrt{4\pi}.$$

Since a volt is 1/300 electrostatic unit, the velocity corresponding to a volt is 0.006 cm. per sec., or about 20 times the velocity of the hydrogen atom when electrolyzed in aqueous solution.

We might naturally look for an effect upon the velocity of light in the direction of the lines of electrostatic force, since F is a velocity. Such an effect was discovered by Kerr, who has shown (Phil. Mag., April, 1894) that only the component vibrating in the plane of the lines of force is changed in the phenomenon which he discovered, the component perpendicular to them being unaffected, but the phenomenon is complicated here by the presence of matter, the effect is so large that it cannot be due directly to the flow of the free ether, and does not vary directly as F.

Rayleigh has examined the effect when light is passed through a solution carrying a current, without observing any increase however, as might be expected, since one volt per cm. corresponds to a velocity of only 0.006 cm. per sec. and the velocity of light is 3.10^{10} cms. per sec.

We can, however, without any extraordinary difficulty, obtain a

voltivity of 150,000 in a vacuum tube, and this would mean a velocity of 800 cms. per sec., which, I learn from my colleague, Professor Wadsworth, should be observable with the refractometer. The experiment will be made.

It may be mentioned here that in doubly refractive substances the piezo-electric effect along different axes should vary with the refractive indices.

To consider next electromagnetic phenomena. We must first find the quality of the curl from the following two equations, representing the effect of a varying electrostatic flux or an electric current, and of a varying magnetic flux :

$$\text{curl}_x\, Q/T = G = H \times L_y \text{ and } \text{curl}_y\, P/T = E = F \times L_x$$

whence

$$\text{curl}_x \text{ (curl round } X \text{ axis)} = L_y/L_x$$

$$\text{curl}_y \text{ (curl round } Y \text{ axis)} = L_x/L_y.$$

It is first to be noticed that a moving flux and a current are not identical in effect. A rotating magnetized ring produces no voltage of the first order of effect in a coil surrounding it. Nor can a rotating glass ring with an electrostatic flux passing round the torus, magnetize to any degree an iron ring encircling it. The movement of the electric lines in a lateral direction is what gives the gilbertance and the lateral movement of the magnetic lines the voltage. Longitudinal motion of the flux has no effect, as there must be a change in the amount of the enclosed flux.

Just outside a wire carrying a current the lines of voltage are almost normal to the surface of the wire, as was pointed out by Heavyside, who discovered the true method of establishment of current in a wire. So long as the ends of the lines are stationary with reference to the wire, there is no current and no magnetic force. But when they move, the original velocity along the lines has compounded with it the lateral velocity of the lines, and the acceleration of the ether measures the value of H at any point. When we establish an electrostatic field between the plates of a condenser, the flow between the plates causes, in some manner, a tension normal to the surface of the plates and a lateral pressure. Whilst charging the plates we are increasing the lateral pressure. This creates a shear in a direction at right angles to both tension and pressure, just

as, in thrusting a ball sidewise between two other balls, the side pressure thrusts the two balls apart and a force at right angles to the lateral pressure is thus produced.

To some extent the above description is founded upon assumptions. For example, it does not follow that because F is a velocity and because it is the velocity of the ether (since we can have a value of F in a vacuum), that all the ether in a cubic centimeter of space is moving when we have a voltage there. This however does not affect the *nature* of the quantity but only the configuration.

I have not suggested any detailed theory of the configuration, for the reasons given in the preface. There are a number of experiments to be made before we can go further with any degree of certainty. Any detailed work on configuration would imply a theory of the nature of matter, which is foreign to my subject. We may of course choose some configuration to work with. In doing this the first thing to be looked after is to have the flux of ether between the two charged plates of a condenser return again, as in a closed circuit. True there is no real reason for this. Continuous flow, without return, is inconceivable, but that is not a very serious objection.[1] To avoid having sinks, however, we may choose some suitable configuration. Possibly the best available is that given by Fitzgerald in his paper "On a Hydrodynamical Hypothesis as to Electromagnetic Actions."[2] According to this, the electrostatic line is a spiral vortex. This gives us our tension between the plates, and the lateral pressure. It can also give us the magnetic shear, and in the paper referred to it is shown that the equations for wave propagation and energy in such a medium as described are identical with those of light. We may also take the atom itself as a spiral vortex, and can get a secondary stress, depending upon angular velocity, to account for gravitation, which would be the gravitational stress. As we have found the rigidity of the ether to be

[1] It is not many years since one of our greatest and most esteemed philosophers (Herbert Spencer, First Principles) came to the conclusion that the idea of an indivisible atom was inconceivable. As a matter of fact it *was* inconceivable then, but it is so no longer. One is reminded of Beaconsfield's witty note to his allegorical history of England (Isle of Fantasia, Chapter IV.) "First principles are the ingredients of positive truth. They are eternal and immutable, as may be seen by comparing the first principles of the eighteenth century with the first principles of the nineteenth."

[2] Royal Dublin Soc., December 12, 1898.

6.10^{20} this would permit of even the large values which Maxwell has pointed out are necessary with a stressed medium theory of gravitation.

Under these circumstances we might expect the permeability of a material to vary with a change in the force of gravity, and a charged condenser to weigh more than an uncharged one. I have not calculated whether the effect should be measurable or not.

In a work extending over many years I have naturally incurred indebtedness on many sides ; I have more especially to express my gratitude to my present colleague, former assistant and sometime pupil, Professor S. M. Kintner, without whose invaluable constructive and experimental ability I should have been able to accomplish but a small portion of the needed experimental work. I have also to thank two other of my colleagues, Dr. Brashear and Professor Frost, for their kind help in many ways. To Helen M. Fessenden also I am indebted for much of the work done in reducing observations and in the preparation of tables.

APPENDIX A.

INVESTIGATION OF THE RELATION BETWEEN μ AND H.

It is evidently desirable that the correctness of this relation should be established with some degree of precision.

It was first put forth by Kennelly in the *Trans. Am. Soc. Inst. E. E.*, Oct. 27, 1891, and has been confirmed by Steinmetz, in a most elaborate and valuable paper on magnetism.[1]

It has also been verified by the author, in a research conducted expressly for that purpose, the method and results of which are given below.

Frölich's formula is $1/\mu = c + dH.$

Kennelly's formula is $\{1/(\mu - 1)\} = a + bH = \nu_i.$

The difference between the two is that Kennelly considers, as mentioned above, the induction contributed by the iron as separate and distinct from that contributed by the ether, in the same manner as, in Gladstone's law, we consider the refraction contributed by the matter as distinct from that contributed by the ether. There are thus two magnetic circuits in parallel ; one, the intrinsic circuit, carrying a flux numerically equal to $B - H$, the other the extrinsic circuit, carrying a flux numerically equal to H. As they have both the same difference of magnetic potential across them, the magnetic resistivity, or reluctivity, is, in the two cases, numerically $H(B - H)$ and H/H.

Kennelly's law means that this intrinsic reluctivity, ν_i is given by the formula $a + bH$, or in other words, that this intrinsic reluctivity varies as the space rate of drop of magnetic potential H varies.

[1] Trans. Am. Inst. E. E., Sept. 27, 1892.

Frölich's formula fails for high magnetizations. Kennelly's does not. Kennelly's formula gives a constantly increasing intensity of flux, with increasing H. Frölich's formula gives the intensity as reaching a limit. This former appears from Ewing's experiments by the isthmus method to be the correct statement.

As will be seen from the papers of Kennelly and Steinmetz, the agreement of the formula with theory is fairly close in all cases examined. It had however been stated [1] that it was a purely empirical formula and was not the expression of any physical law. The writer, therefore, made a set of tests with a view to obtaining results of as high a degree of accuracy as might be possible.

In taking curves of magnetism, the following points are important:

1. Long wires cannot be used, as the leakage varies with the reluctivity, and the area of leakage surface is relatively large compared with the cross section. At low values of μ the results may be many per cent. out.

2. Rings wound from wire cannot be used, as the magnetic flux has to jump from one wire to another through an air gap, and thus an additional reluctance is introduced, and moreover the inductance is not uniform, and the true shape of the curve is greatly masked. Steinmitz has shown[2] that when the flux has to pass in this way the variation from the mean induction may be so great as to increase the mean hysteresis 70 per cent. in sheet iron, and no less than 900 per cent. in iron wire, above the hysteresis corresponding to the mean induction.

3. Rings punched out of sheet iron are the only ones permissible, as it is not possible to weld a solid iron ring and keep its constitution and fiber uniform. Moreover, magnetic creeping is worse in solid rings.

4. The outside diameter of the ring must not be more than ten per cent., preferably not more than five per cent., greater than the interior diameter. In a ring of interior diameter 6 inches, external diameter 10 inches, which the writer once saw used for the determination of a permeability curve, it is evident that when H was 3 on the outside circumference of the ring it was not less than 5 on the inside circumference. Consequently, when the outside portion of the ring was on the downward part of the reluctivity curve, the inside part was on the upward side, and the curve obtained from such a ring must be entirely misleading, the features of the curve being run together, and the true reluctivity being at one point much lower than it would appear to be from the test.

This fact, that the diameter of the ring may affect the results by five per cent. or more, has been pointed out previously by Kennelly.

5. Rheostats with sliding contacts cannot be used for this work, as they give very insidiously distorted curves. This is due to the fact that if we are at P, Fig. 6, and the resistance in the circuit is, say, 200 ohms, and the next resistance 400, then on moving the contact the resistance does not change from 200 to 400, but from 200 to, say, 800; and from 800 to 400, due to the fact that when moving from one section to another the contact is not very good, and though of course not necessarily broken with a properly designed rheostat, yet the contact is worse when moving from one section to another than when at rest.

The result is, curves of the shape shown in Fig. 6 in the dotted line, where E, after sinking down lower than it should, comes back to a lower point than it would otherwise have fallen to. The result is a flattening, or even a concaving of that part of the curve.

A similar thing occurs on the up curve, and on the plain induction curve. The distortion is small, but since the curve is very steep, it makes a large percentage error in calculating the ferric reluctivity.

[1] Elec. World, June 9, 1894.

[2] On Hysteresis, Trans. Am. Inst. E. E., 1892, pp. 699 and 701.

6. All throws of the galvamometer must be of the same magnitude. This because the law of the galvanometer cannot be depended upon, the theoretical laws being only approximately followed and not closely enough where an accuracy of $\frac{1}{10}$ per cent. is required. A much more important reason is that the amount of creeping seems to be approximately proportional, according to some experiments of the writer's students, to the change of induction. It is therefore evident that, by changing B by a constant amount each time and taking approximately the same time for each throw, the results will probably be more reliable than when working more or less at haphazard.

7. If we change H to H' and then B to B' the state reached at B' will not be stable, even after creeping has stopped.

For if we wind on the iron ring a few turns of wire through which a very feeble current may be sent, which we call dH, then, on changing H from say 3 to 4. and then observing the value of B, if we next throw on dH so that its gilbertance is in the opposite direction to that of the main coil, we shall find that B is diminished by, say 50 lines. If, however, the direction of dH is the same as that of H, then B will be increased by, say 5 lines only. This would be bad enough, but the matter is still further complicated by the fact that if, after changing H from 3 to 4 and then throwing dH on in the same direction as F, giving an increase of from B to $B' + 5$, we then reverse dH, B' does not fall, as might be expected from $B' + 5$ to $B' - 50$, but only to $B' - 5$. What the ultimate value of B' is, then, when $H = 4$ and dH has been removed, depends upon which direction dH was first applied.

It is, therefore, impossible to get accurate magnetic curves with a dynamo current, as the minute fluctuations affect the result, and may not affect it twice in the same way.

This effect was investigated by one of the writer's students, Mr. Davis,[1] in 1896–7. The amount of the effect seems to depend upon the permeability. For this reason the step by step method was used in preference to that of reversals.

An additional reason for using a battery current is that small changes of H due to small pulsations in the current, as the brushes pass over the commutator segments, seem to change the rate of creeping, and also the amount.

The existence of the above phenomena is an additional reason against using a rheostat with sliding contact.

8 The time constant of the circuit must be small as compared with that of the galvanometer, as otherwise the system will begin to move before all the discharge has passed. Sometimes the time constant is larger than would be expected without calculation.[2]

In the test about to be described the above effects were taken into account, as well as others, better known, due to thermo-electric currents, leakage, heating of resistance, and lack of uniform temperature.

The rings were of the best kind of transformer iron, cut to size on a tinner's machine, not punched, and carefully annealed. They were kindly furnished by the Westinghouse Co. In other samples, where the method of annealing was such that gas or carbon might be absorbed in the annealing, I have found that good results were obtained by enclosing the rings, embedded in sulphur-free iron peroxide, in a cast-iron box, with sides one inch thick. Some of the peroxide will be reduced, and it can be seen whether the gas has got to the plates or not by the difference in color of the oxide.

[1] Thesis, "On the Closed Magnetic Circuit in Telephones."

[2] In the case of the field rings for the Niagara generators, the time constant was measured in minutes, and Mr. Scott and the writer had, therefore, to devise a method in which the time rate of change of induction was kept constant, and the time for the complete discharge was the thing noted.

Each individual ring was examined to see if the coating of oxide was abnormally thick or held much magnetic oxide. If so, it was rejected. So far, the writer has been unable to find any way of getting at the true cross-section of the iron without destroying the rings in cracking the scale off, but for the present purpose that was unnecessary, as relative values only were required and a mistake as to the cross-section would affect all constants in the same ratio. The accepted rings were measured across three diameters, inside and outside, on the bed of a comparator.

This may seem an unnecessary refinement, but it was the shortest way, and it was thought best to know the mean circumference with some degree of accuracy.

The rings were then shellaced, dried, built up into one thick ring, 10 inches inside diameter, 12 inches outside diameter, and 1 inch thick. This was then carefu ly insulated with varnished paper, and both primary and secondary wound symmetrically, *i. e.*, the wires spired evenly round the whole of the circumference so as to prevent uneven distribution of magnetic leakage. Then the whole was shellaced and dried.

The galvanometer was astatic, of a well-known type, with four bell-shaped magnets in the system, being rendered more or less sensitive by screwing up a ring on one of the magnets.[1]

It was enclosed in a cylinder of soft cast iron, with top, sides and bottom of a thickness of 6 inches. The mirror was true, and the cover glass of the galvanometer was a parallel plane, ground by the writer at Mr. Brashear's shop.

The scale had been tested by the Société Genevoise comparator mentioned above. The telescope was of the usual type, and special arrangements were made to secure good lighting of it.

The suspension was a quartz fiber, 6 inches long. The period of the system was 20 seconds. reached by laying a piece of a magnetized file inside the iron case between the bottom of the case and the bottom of the galvanometer and moving it about until the desired sensibility and position were obtained.

The variable resistance was made after the fashion of a Varley rheostat, two wires wound in grooves on two pairs of wooden cylinders 1 foot in diameter. But along one cylinder of each pair was laid a metallic strip with holes drilled in it. In these holes small copper plugs could be screwed, in such fashion that where there were no copper plugs the wire passed over the strip, but where a plug was inserted the wire rested on it and so made contact with the strip. In turning the cylinder the wire was wound up until it reached the groove where there was a copper plug, when the contact was made and the resistance of the circuit decreased by the amount desired. In hysteresis tests, one pair is used for the up curve and one for the down.

A preliminary run was made between the values of H desired, with the plugs arranged by guess. From this the position in which the plugs must be placed to give equal deflections on the galvanometer were calculated, and the plugs so placed. The wires were German silver, of ample carrying capacity, and the heating very light owing to this and to the large surface exposed to radiation. The ends of the pins were grooved so that the contact should be good and sharp.

The iron was demagnetized by careful reversal, in diminishing cycles, combined with tapping, the ring being hung so that the plane of the ring was perpendicular to the direction of the earth's total magnetic field. The ring at first had an alternating current generated in it, with a view to shaking out the magnetism,[2] but this was afterward abandoned, as it was thought that it might introduce errors by possibly leaving some circular magnetization. The current was measured by a Weston milli-

[1] Willyoung, Elect. Eng., N. Y., Nov. 16, 1892.

[2] Ewing, Mag. Ind. in Iron, p. 318.

ammeter which could be read to $\frac{1}{15}$ per cent. of the maximum magnetizing current used, *i. e.*, about 1.45 ampères. The proportionality of this scale had been corrected for a particular position and orientation in the laboratory, by an accurately calibrated slide wire bridge, almost free from thermo-electric effects. Its absolute constant was determined by a Kelvin centi-ampère balance, whose constant had been frequently determined, but not much care was taken with this, as only relative values were desired and any great accuracy as regards this point would indeed have been absurd, in view of the fact that the true cross-section of the iron could not be accurately known within several per cent., so that absolute values were unobtainable.

The ring was suspended in slings in a wooden frame, to prevent vibration and allow free radiation, and placed so as to be free from all outside magnetic influences. The temperature of the galvanometer, ring and ammeter were known. The measurements were made in the spring of the year, and during the hour which each set took, approximately, the temperature of the basement room did not change, as a rule, more than two degrees, whilst the galvanometer temperature change was so slight (in its double covering), that it never reached $\frac{1}{10}$ degree, which was as small a change as the thermometers used could show. The wire used on the iron ring, consisting of flat strands of fine wire for both primary and secondary was of ample cross-section, for the double purpose of keeping the heating negligible and of having that part of the resistance outside the galvanometer box as small as possible, so that changes in the room temperature would have but a small effect on the total resistance of the secondary circuit.

In making the test, approximately the same time elapsed between each throw. Each throw was about 15 inches, and as the inches were divided into tenths and each tenth could be read to the decimal part, the accuracy of the reading must have been approximately $\frac{1}{10}$ per cent. The constant of the galvanometer was determined, roughly, as accuracy was not necessary here, by a mutual induction coil, wound on glass. By means of this coil and the Weston milliameter mentioned above, the relation between quantity and throw was very accurately determined, especially for that part of the scale which the readings obtained in the experiment proper fell upon.

The third and best curve obtained is given in Fig. 8. The dots representing the observations are covered by the line, the greatest deviation from the line being almost exactly $\frac{1}{8}$ per cent. of the maximum ordinate, when all corrections were made. Below $H = 2\frac{1}{2}$ the line begins to deviate from the formula, and below $H = \frac{1}{2}$ the agreement is not even approximate.

As a further example of the accuracy obtainable by this method it may be said that a set of six hysteresis curves was taken, in the following year, by Mr. Kennelly and the writer in conjunction, with the same apparatus (with the exception of the resistance cylinders, for which were substituted separate coils hanging in mercury cups and suspended in mineral oil, being short circuited or cut in circuit, as it was desired to vary the current) and, with much more unfavorable temperature conditions, the worst hysteresis curve obtained closed up to within $\frac{1}{4}$ of one per cent. of the difference of B between $H = +$ max. and $H = -$ max. the best curve being out $\frac{1}{10}$ per cent., and the error of the other curves lying between these values. Without the above mentioned precautions the writer has never been able to *depend* upon hysteresis curves closing up within two per cent.

The curvature at the commencement is always found. It becomes much more sharp when the difference between the inside and outside diameters of the ring is taken into account.

If not, the error may amount to several per cent., and, in fact, for accurate results this must always be allowed for.

In the curve given, Fig. 7, a curve was drawn first from the corrected observations.

Then from the values of a and b obtained from this first curve, an allowance was made for the error caused by the mean permeability not being the permeability of the mean length. From this the correct curve was drawn, it not being necessary to make a second approximation, as this would alter the values by less than the experimental probable error.

The elbow may be removed, as shown in the following curves, Fig. 5, taken from the observations of Gerosa and Finzi,[1] which show that when the iron is shaken up the elbow disappears, but that a and b are not greatly changed.

It is thought that the above experiment, made with a view to testing the matter as accurately as might be, proves rather definitely that we have here no rough empirical formula, but the expression of a physical fact; *i. e.*, that the intrinsic reluctivity is a linear function of the drop of magnetic potential per linear cm. This may be considered the touchstone of the electrical theories, and as proving without doubt that κ is a density and μ the reciprocal of an elasticity.

The following formulæ are given, as being of use in electromagnetic design:

$$(1, 2, 3)\quad \nu_i = a\Sigma_0^{\alpha} bB = a + bH = a/(1 - bB).$$

$$(4)\quad B = H/(a + bH). \qquad (5)\quad H = aB/(1 - bB).$$

$$(6)\quad .4\pi nI = P\left[\frac{aL'c}{R - bPc} + \frac{a'L'c'}{R' - b'Pc'} + \cdots\right].$$

$$(7)\quad \frac{dB}{dH} = a/\nu_i^2 = aB^2/H^2. \qquad (8)\quad \frac{b\eta}{a} = .007 \text{ approx.}$$

Where B has been, for practical purposes, taken as equal to B_i, in nos. 4, 5, 6 and 7. I represents current, n = number of turns, P = total magnetic lines, B = lines per sq. cm., H = difference of magnetic potential per cm., $\nu_i = 1/(\mu - 1)$, L = length of part of magnetic circuit considered, R = cross-section of same, c = leakage coefficient of same, η = hysteresis loss per sq. cm. per cycle.

Appendix B.

Résumé of Previous Work.

It was pointed out, in another portion of this paper, that given a knowledge of the nature of the electric and magnetic quantities, there is no difficulty in inventing suitable theories in regard to configuration, the number of such theories being chiefly a matter of permutations and combinations, but that each theory of the configuration virtually implies a theory of the constitution of matter.

In the present appendix it is not my intention to give an account of all the work done during the search for a solution of the problem with which this paper is concerned, for a record of this work, which has occupied a considerable portion of my leisure during a period of ten years, would be very voluminous. I shall deal only with that part of it which tends in some way to throw light upon the nature of the atom, and with that but very briefly. The results will be given in logical order and not in the order in which they were obtained.

Properties of the Atom.

Various estimates have been made of the size of the atoms. There is, however, this question to be first answered: "Have the atoms definite sizes?"

If we take 45 c. c. of potassium and 18 c. c. of solid chlorine they can unite chemically to form a quantity of KCl occupying a space of but 37 c. c. One natural explanation of

[1] See Ewing, Mag. Ind. in Iron, p. 319.

this fact that has been suggested, is that even in a solid the atoms do not lie closely adjacent to each other and hence there is abundant space for contraction even after the addition of other atoms. But this appears to be negatived by a number of facts. In the first place it is difficult to see how fusion can mean anything else than that the atoms have been driven so far apart by reason of the increase of kinetic energy that they can slide over one another and take each other's places. Hence in a solid it seems reasonable to suppose that they are so close together that this sliding over is impossible. Secondly, from Van der Waal's equation we may make an estimate of the volume of the atoms compared with that of the space occupied by them at the critical volume, and, hence, if we further deduce what this ratio must be for the solid state we see that the atoms must be nearly touching each other in this latter condition. Again, the fact that the electric conductivity and some other properties vary directly with the temperature as the temperature is lowered whilst these same properties are much altered by change of density, renders it hard to believe that there is any abrupt change in the volume of solids as we approach the absolute zero, which would apparently be necessitated by this explanation.

A second theory is that of Shröder, who assumes that the atom may have two or more different volumes, the volume of the atom depending upon the nature of the chemical combination.

It has been shown by the writer [1] that neither of these suppositions is necessary. If we take 100 balls of pitch,[2] each having a diameter equal to that of a cube containing .45 c. c. and stack them in rectangular array, as shown in Fig. 1, *a*, they will occupy a space of 45 c. c. We may assume the rectangular order as that naturally taken by bodies

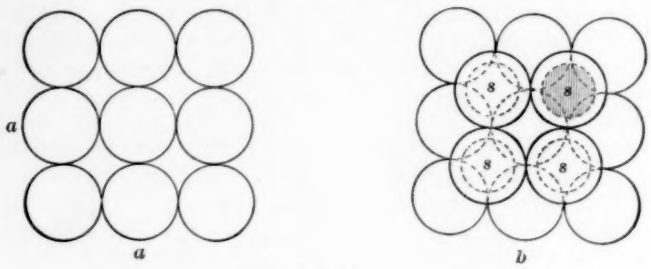


Fig. 1.

attracting each other and at the same time repelled by their mutual collisions. If the repelling force be increased by further heating, they separate further and can now slide over one another. If the kinetic energy be lessened, or some additional attractive force brought into play, the atoms come closer together and the substance is no longer isotropic but tends more to be crystalline.

Take now 100 other balls, each having a diameter equal to the side of a cube whose volume is .18 c. c. Stack them in rectangular order and they will occupy a space of 18 c. c.

Next arrange alternate layers of the large and small balls, as shown in Fig. 1, *b*, where the attraction of the chlorine atoms is supposed to have pulled the potassium atoms over,

[1] Elect. World, Aug. 22, 1891. Science, March 3, 1893.

[2] Best made by pouring a slab of pitch, cutting up to *weight* and rolling between boards covered with pigment. Or, if a squirting machine is available, by cutting up squirted rod in proper lengths and rolling as before.

as shown. On measuring the volume now occupied by both sets of balls it will be found to be nearly 37 c. c.

This simple experiment may not be considered convincing. But when we find, on applying the simple formula thence derived,[1] that we obtain not only the correct molecular volumes of KCl, NaCl, KOH, NaOH, Cæs. Cl, Rub. Cl, and other salts, and when in addition we find that in most cases so far tested the experiment also gives the correct crystalline angles,[2] we are to a certain extent justified in holding that

A. The space occupied by the atoms of a metal to the exclusion of other atoms is of an approximately spherical shape at ordinary temperatures.

B. The atoms of a metal, the metal being in a solid state, are not widely separated, but the shortest distance between the surfaces of two adjacent atoms is of the order of one per cent. of the diameter of the atom itself, when at temperatures not near the fusion point.

Metalloids, as we know from their low specific heats, form molecular groups in the solid state. Hence we may have more than one melting point, first when molecule can slide over molecule, and second when atom can slide over atom. If the atom or molecule be very irregular in shape, a separation sufficient to permit of this sliding may so weaken the cohesive force that it is overbalanced by the kinetic repulsion and the substance gasifies without melting.

Since also we find that it is possible to predict the crystalline shape of but few of the metalloids by means of the method mentioned above, we are led to consider the following statements as probable. For definition of metalloids see Fig. 2.

C. Metalloids differ from metals in that their atoms have not an approximately spherical shape.

D. The melting point and expansion of the elements is a function of the shape of the atoms.

Cauchy, in 1835 made the first estimate of the size of the atom, and in 1870, Kelvin by other methods arrived at the conclusion that the mean distance between the centers of contiguous molecules was less than 10^{-8} cm., and more than 5.10^{-10} cm.

Recent determinations by different methods agree quite closely, and it would seem as if the mean of the results cannot be far from the truth.

Given by.[3]	Value.
Nernst, Theoretical Chemistry	2.58×10^{-8}
Nernst, " "	2.69×10^{-8}
Risteen, Molecules and Mol. Theory	3.8×10^{-8}
J. J. Thomson, Recent Papers	3.34×10^{-8}
R. A. Fessenden, unpublished	2.79×10^{-8}

Taking into consideration the direction of the probable error of these determinations we obtain :

E. The diameter of the mercury atom is $2.75\ (\pm 0.2)10^{-8}$ *cm.*

F. The ioni charge is $4\ (\pm 1)10^{-10}$ *E. S. units.*

This ionic charge, " the natural unit of electricity," as Lodge aptly calls it (first calculated by Everett, and later, independently by Lodge) has long been considered as the cause of chemical action. This theory has been worked out in considerable detail by

[1] Science, March 3, 1893. [2] Molecular Physics, Frank. Inst., September, 1896.

[3] Nernst, from Clausius-Mossotti theory of dielectrics and from kinetic theory of gases; Risteen from surface tension of mercury by Kelvin's method; J. J. Thomson from ionic equivalent of gases; Fessenden from surface tension of mercury by Ostwald's method. All reduced to mercury atom.

Davy, Berzelius, Helmholtz, Arrhenius, Clausius, Ostwald, Nernst and others, with the result that we no longer consider "chemical force" as a separate force but merely as a particular manifestation of electric force.

The writer was able in 1890 to make a still further extension of the field of electric action by showing that there is no necessity for assuming a separate "cohesive force,"

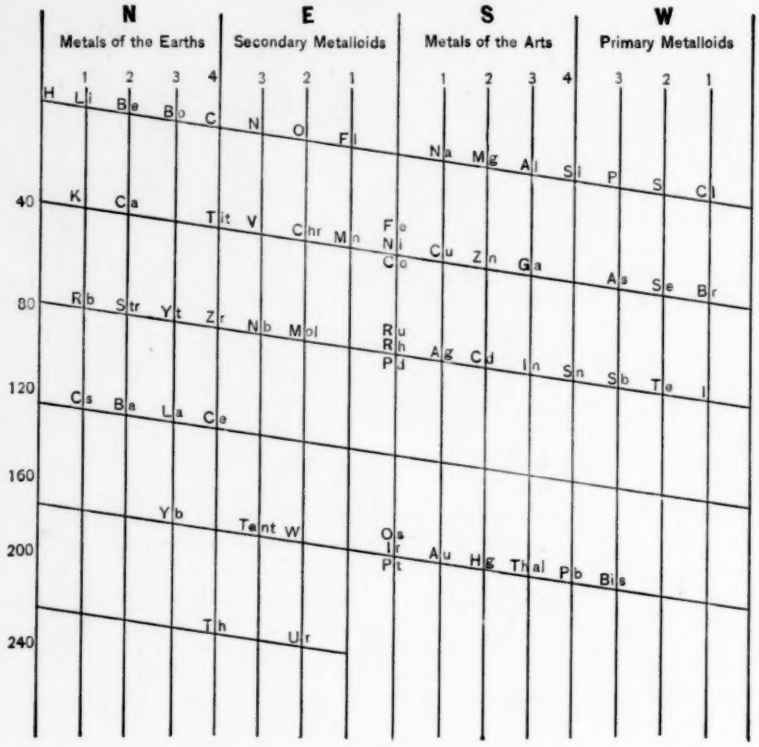


Fig. 2.

but that the ionic charges fulfilled yet another distinct function, in that they are capable of accounting for the tensile strength, rigidity, Young's modulus and other mechanical properties of bodies. The field of physics, by the removal of these two forces, has thus been reduced to the consideration of the forces connected with electricity, magnetism, gravitation and inertia.

According to this theory,[1] the normal atom, in the neutral state, is really a doublet, *i. e.*, has two equal charges, one positive and one negative. Two such atoms when at a distance will exhibit no attraction for each other, but on being brought near one another will be attracted. Attraction under these circumstances we take to be cohesion. "If however, any third substance" (or as was pointed out later, any other molecular form of the same substance) "connects the outside parts of the atoms and so enables these parts

[1] Not published till July, 1891, as the then novel idea of the atoms of metals having charges was considered by the editors to whom it was submitted as incompatible with the law that all charges must reside on the surface of conductors.

to neutralize each other, we have chemical combination, and the two parts when separated show opposite charges, as at C."[1]

According to the electrostatic theory of cohesion, therefore,

G. Chemical action and cohesion are both manifestations of the same force, i. e., electrostatic attraction, under different conditions.

The resultant product of a cohesion effect is generally isotropic, whilst that of a chemical action (owing to the presence of atoms of different sizes or valencies) is crystalline, *i. e.*, "the atoms in one case may be grouped in any direction, and in the other they are only grouped about certain axes or planes."[2]

Taking the ionic charge as 4.10^{-10} E.S. units and the diameter of the silver atom as $2.42\ 10^{-8}$ cm., we find that the total quantity of electricity on a sheet of silver one atom thick and one cm. square is $6.8\ .10^{5}$ E.S. units. If for a first approximation we suppose the flux to be uniformly distributed we find for the attraction between two such sheets 3.4×10^{7} dynes.

Considering a silver wire one mm. thick as made up of discs one mm. in diameter, we find, on this theory, that the attraction between two such discs is 270 kilogrammes.[3]

From Wertheim's experiments the tensile strength of a silver wire one mm. in diameter is 29 kilogrammes.

We have taken the flux density as uniform. We know that this cannot be true or the resistance to lateral shear would be very slight, in fact the substance would be a fluid. The amount of the attraction will depend upon the distribution of the charges, but we will find, on examination, that if we reject the idea of indefinitely large flux densities at any point, any conceivable distribution will give results of the same order of magnitude as those given above.

In this calculation we have neglected the kinetic repulsion. Our justification for this is that since the kinetic repulsion varies roughly as the cube of the distance between the surfaces of the atom, and the attraction varies as some power less than the square of that distance, the force required to stretch, shear or pull apart a body will, to a first approximation, be determined by the attractive force alone.

If the theory be correct and the disposition of the charges does not vary greatly for atoms of different elements, the mechanical properties should be functions of the size of the atoms, since the ionic charge is the same for all atoms. The following formulæ and tables show that such is the case:

A. Rigidity $= 28 \times 10^{12} \div (\text{atom. vol.})^{2}$

B. Young's Modulus $= 78 \times 10^{2} \div (\text{atom. vol.})^{\frac{2}{3}}$

C. Velocity of Sound $= \{78 \times 10^{12} \div (\text{atom. weight} \times \text{atom. vol.})\}^{\frac{1}{2}}$

D. Tensile Strength $=$ Absolute Temp. of Melt. pt. $\div (1.92 \times \text{atom. vol.}^{\frac{1}{3}})$
The last is for wires 1 mm. in diameter.

The elasticity of such substances as India-rubber, gelatine and ivory was investigated. It was found that the abnormally high value of Poisson's ratio for such substances was due to the fact that their elasticity, as measured in the ordinary way, bears no direct relation to the true elasticity of the material, but is a matter of configuration. It was shown[4] that

[1] Science, July 22, 1892. J. J. Thomson showed later that the proximity of a dielectric of greater capacity would have the same effect.

[2] Molecular Physics, Franklin Inst., Sept., 1896.

[3] Given as 45 kilogrammes in *Electrical World*, August 22, 1891, owing to a slightly different value having been taken for the size of the atom.

[4] Mol. Phys., Frank. Inst., September, 1896.

such substances consist of two or more compounds, interpenetrating but not combining with one another. It was shown that the heating on stretching, the cooling on relaxing and the shortening when stretched and heated, followed as a natural consequence of the configuration. The different compounds were differentiated under the microscope and, in the case of India-rubber, by following Faraday's method, isolated chemically. The shape of the elasticity curve was predicted and the prediction was verified by the writer and by Professor Thurston.[1] Substances were made which had the shape of the elasticity curve, the property of polarizing light on being stretched, and the value of Poisson's ratio the same as rubber, but which on simple pressure could be differentiated into two substances, each of which had the usual form of elastic curve, now heated instead of cooling, on compression, and now had a value of Poisson's ratio nearly one-half of what it was previously. The behavior of these abnormal substances was thus shown not to conflict at any point with that called for by the electrostatic theory of cohesion.

TABLE I.

Metal.	Atom. Vol.	Rigid. $\div 10^9$	Calc.	Young. M. $\div 10^9$	Calc.	Tens. str.	Ca
Iron.	7.1	750	550	2000	1560	65	74
Copper.	7.1	430	550	1220	1560	41	48
Platinum.	9.1					35	48
Zinc.	9.2	350	340	930	920	15.7	16
Silver.	10.2	280	270	740	750	29.6	29
Gold.	10.2	270	270	760	750	28.5	29
Aluminum.	10.4	250	260	680	690	18.	18
Cadmium.	13.		170	480	465		
Magnesium.[2]	14.	150	143	390	395	10.4	9
Tin.	16.3	136	100	420	295	3.4	5
Lead.	18.1	84	83	190	235	2.36	4

It was found that there existed a relation between the velocity of sound and the electric conductivity of the pure metals as follows:

II. If we take wires of two pure metals, having an equal number of molecules in the cross section, then the resistances of the wires will be proportional to the times taken by a sound wave to traverse them.

The supposition is here made that whilst, as regards specific heat, the pure metals behave as if there were only one atom in the molecule, as regards electricity they behave as having the number of atoms per molecule numerically equal to the valence. The grounds for this assumption may be questioned, the reasons for making it are given in another paper.[3]

The formula for the electric resistivity at 0° C. of any metal is thus:

$$\text{Resistivity} = 14.04 \times \text{atom. vol.} \times \sqrt{\text{atom. weight}} \times \text{valency}.$$

Table II. on opposite page shows the observed and calculated resistivities.

The tests on Zn, Sn and Pb were made on pressed wires.

It should be pointed out that the conductivities of two substances, which, as determined at the time when this relation was first published,[4] were too low, have since

[1] Science, May 1898.

[2] The tensile strength of magnesium was predicted, Science, July 22, 1892, and Thurston's tests, London Elect., June 5, 1896, showed it to agree with theory.

[3] Conduction and Insulation, Amer. Inst. E. E., March, 1898. [4] Science, July 22, 1892.

been found, by later experimenters working with pure materials, to be in agreement with the formula. I refer to aluminium and magnesium, whose resistivities were given as 2881 and 4162 respectively, but the former of which has been found by Richards and Thomson [1] to have the restivity 2370 for the pure material, and magnesium, which has been shown by Thurston [2] to have a resistivity of 3835. In the case of zinc, tin and lead it will be noted that through the absolute values are low, the ratios are correct. It is barely possible that the fact that these three materials were all tested in the form of squirted wires may have something to do with the discrepancy.

TABLE II.

Metal.	Atom. vol.	Atom.wt.½	Valency.	R. calc.	R. obs.	Observer.
Cu.	7.1	7.94	2	1583	1580	Mathiessen.
Ag.	10.2	10.39	1	1488	1488	"
Au.	10.2	14.05	1	2012	2036	"
Al.	10.5	5.2	3	2300	2370	Rich. & Thomp.
Mg.	14.	4.9	4	3852	3835	Thurston.
Zn.	9.4	8.09	4	4270	5566	Mathiessen.
Cad.	13.	10.58	4	7722	6720	Benoit.
Sn.	16.2	10.86	4	9880	13070	Mathiessen.
Pb.	18.2	14.38	4	14695	19420	"

This formula, whilst in some cases not entirely satisfactory (though in view of the fact that already in two cases the fault has been found to lie with the physical measurement and not with the formula, it may happen that the other exceptions may also fall into line) is of interest as linking together the phenomena of the conduction of sound, heat and electricity in wires. I have suggested elsewhere that the velocity with which heat and electricity travel in a wire may be that of sound. In the case of heat, the experiment might possibly be made, in spite of the very rapid logarithmic tailing down, due to the wire's heat capacity (hence similar to the case of an electric cable), by a thermocouple at one end of a short piece of gold or platinum, heated very suddenly at the other end by electric means. But in the case of the electric current, as Maxwell and Heavyside have pointed out, we know absolutely nothing of the velocity with which electricity travels in the wire. "It may be an inch an hour or it may be immensely great." [3]

The theory was applied to fluids. It was shown that whether a body is solid or fluid or a gas or vapor depends upon whether the fraction

$$\frac{\text{Cohesive force of atoms for one another} + \text{external force}}{\text{Kinetic repulsion} + \text{cohesive attraction for other atoms}}$$

is greater or less than unity. From this equation of state it will be seen that we can produce dissociation in two ways: by decreasing the numerator or increasing the denominator. If we increase the first term of the denominator we disassociate by heating, if we increase the second term by bringing the atoms, say of NaCl, in contact with other atoms, of say H_2O, we disassociate by solution. It is shown that this leads us to the idea of an "osmotic suction" instead of an "osmotic pressure," and that the change in the

[1] Journal Franklin Institute, March, 1897.
[2] London Elect., June 5, 1896.
[3] Heavyside, Papers, II., p. 394.

"volume force" thus brought about accounts for the heating on solution, and other difficulties of the "osmotic pressure" theory. Some applications were made to the theory of electrolytic conduction.

The theory was also applied to gases, and it was pointed out that the constant a in Van der Waal's equation

$$(p + a/v^2)(v - b) = RT$$

should be a function of the volume b. That this is true is shown by the following table, the values being taken from Ostwald's "Outlines of General Chemistry,"

Substance.	$a \times 10{,}000$	$b \times 10{,}000$.
Dyethylamine	355	58
Ethyl. Acet.	348	55
Ether	324	57
Benzine	(438)	51
Ethyl. Form.	304	48
Chloroform	287	44
Acetone	273	44
Methyl. Acet.	248	39
Alcohol	236	37
Ethyl. Chlor.	227	40
CS_2	219	33
SO_2	123	24
NO_2	(74)	19

and a modified form of the equation, *i. e.*,

$$(p + c/b^{4/3})(v - b) = RT$$

was suggested, c having the same value for all gases.

Although received with so little favor at first that it was over a year before publication could be obtained, the theory seems now to be on a better footing. The electrification of separate sheets of mica, etc., is in its favor, and Ostwald pointed out in a letter to the writer, September 16, 1891, that some experiment of his along a similar line were of interest in this connection.[1]

About a year and a half after its first publication, it was shown by Chattock (Phil. Mag., Dec., 1892) that the piezo-electric and thermo-electric properties of crystals, and the dielectric strength of gases, could be accounted for along these lines.

In the case of these phenomena, since we are dealing with chemical compounds, and not simple elements, there is nothing to show that the effect is not merely a mechanical-chemical one, or that there does not exist a force of cohesion *in addition* to the force exerted by the electric charges which we know to be active in chemical action. But Chattock was led, in addition, independently, it appears, to the same conclusion as the

[1] "The electrostatic theory of cohesion is new to me, and interesting, in as much as, according to my experiments, the cleavage surfaces of almost all bodies appear to be electrically charged." A point which he raises, "For electrolytes there would be the question to answer, 'Why substances like alcohol, etc., show no ions?' whilst according to your theory, all elements have electric charges" is, I think, explained on taking into account the different ways in which the atoms are charged in chemical and cohesive effects.

writer, *i. e.*, that cohesion as well as chemical action is to be accounted for by the ionic charges.

Magnetic Rotation of Light.

Both Kelvin and Maxwell, from a study of the phenomenon of the magnetic rotation of light, came to the conclusion that, to quote Maxwell, "at each point of the medium something exists of the nature of an angular velocity about an axis in the direction of the magnetic force."

I have never been able to persuade myself of the validity of the proof given, as, in spite of Maxwell's disclaimer, it seems to me that both proofs virtually imply a certain class of theories of light. If, indeed, it were proven that rotation is possible in vacuo, then we would be forced to conclude that some one of these light theories was true. But so far this has not been shown.

It is, I think, not difficult to see that the phenomenon may arise from a simple strain, and that an angular velocity is not necessitated. The rotation of a plane wave is reversed on reflection backwards through a suitably chosen medium, but the rotation of other forms of periodic motion may be doubled. Take a couple of wires of equal length, and string beads on them. Twist one into a right-handed spiral and the other into a left-handed spiral. Take any medium such that on stress it becomes a mass of right- or left-handed spirals, for example a lot of strands of wire rope, straightened and packed loosely side by side. In this condition, if equal forces be applied to right- and left-handed spirals, one will pass through the medium as fast as the other. Compress the bundle of strands, however. It will then turn to a mass of loosely packed right-handed spirals. Through this the right-handed spiral will now penetrate much faster than the left-handed. If, after passing through the medium, the wires strike a plate, and being bent back, are forced to reënter the medium and traverse it in the reverse direction, the right-handed spiral will still travel the faster, and its total gain will be twice that given by traversing the medium in the original direction.

So that the assumption that when we have a magnetic field we have an angular velocity about the axis of magnetic force really implies a theory of matter, and taken by itself, in the absence of any knowledge as to whether light can be rotated in a vacuum, is incapable of proving anything in regard to the nature of the magnetic field.

I have pointed out elsewhere[1] that "It might be supposed that the electro-magnetic rotation of light would give us a fourth equation. * * * Since the atoms carrying charges are vibrating in every direction, we have really an infinite number of electric currents, and it is obvious that when such a system is placed in a magnetic field the velocity will be greatest in a direction parallel to the current in the nearest part of the magnetizing coil, as if the motion be otherwise, work will be done in forcing the atoms to move in that direction. In either case we get only an equation between the electric and magnetic quantities which is implicitly contained in the first three equations."

This change in velocity when the ions are placed in a magnetic field has since been shown by Lorentz to be the cause of the Fiévez or Zeeman effect. It was, however, found that this ionic movement did not give results of the same order as those necessary for the explanation of magnetic rotation and the following theory (communicated to Professor Fitzgerald, December, 1898) was taken as being the most probable one.

In the electromagnetic wave the energy exists in two forms—electric and magnetic. Consider the electric displacement by itself. It produces, by itself, a magnetic force which is a maximum at the instant when the displacement is zero. So long as the energy

[1] Electrical World, May 18, 1895.

is being transmitted without loss, the electric displacement and the electric force producing it are a maximum at the same instant. But suppose that there is absorption, then the voltage and displacement current will no longer be 90 degrees apart, but the angle will be less. The voltage may now be considered as split up into two components, one in phase with the displacement and one at 90 degrees from it, in point of time. Hence while, when there was no loss of energy, the plane of the electric displacement was merely rocked to and fro, there will now be a to and fro rocking due to the one component, together with a continuous twist due to the other component similarly with the magnetic component. In other words, whenever we have absorption, the light wave is rotatable when placed in a magnetic field. Some evidence in proof of this has already been obtained, and will be published later. Here it will be sufficient to point out that those substances which absorb most strongly, such as selenium, or copper oxide, give the greatest rotation, and that the rotation appears, in those cases so far examined, to be greatest at that point in the spectrum where the absorption is greatest.

THERMO-ELECTRICITY.

A number of experiments were made to see if there was any evidence that thermo-electricity depended upon the energy of combination of the materials of the circuit, analogous to the discovery of Exner that the voltage of cells could be calculated from the heats of combination of their elements. It was found, after examining the records of a number of thermopiles, that in every case the total amount of energy which had been obtained from them was of the same order as that which would have been furnished by the heat of combination of the materials forming the couples. Two holes were then bored in a slab of antimony, and the two ends of a copper rod of ½ inch diameter were driven in them. The whole was then fixed to a steam pipe in constant use so that one end was in contact with the pipe and the other in the free air. After a year the combination was examined. It was found that at one junction there was a considerable quantity of violet colored alloy of copper and antimony, whilst at the other there were only traces. As, however, one of the contacts had gone bad, at some time unknown, it was impossible to make a quantitative estimation, and the experiment will be repeated at a favorable opportunity. It is possible that thermo-electric effects are intimately connected with the phenomenon of molecular diffusion in solids, discovered by Roberts-Austen, and so strikingly illustrated by his admirable experiments.

CONFIGURATION OF ATOM.

Since every atom is charged and the charge has the quality M/T, *i. e.*, current of inertia, we thus for the first time have a definite proof of the vortex nature of the atom. A considerable number of other points were investigated, more or less fully, but will be discussed elsewhere.

APPENDIX C.

NOMENCLATURE, SYMBOLS AND UNITS.

Mental, as well as mechanical processes, are vastly facilitated by the presentation of the material to be worked up in a form adapted to the machinery which is to be employed upon it. When we have stated our data in terms of suitable units, we have accomplished a sort of preparatory rough planing and surfacing of our facts, and as a result we may then proceed to make our deductions with much greater ease and with greater certainty.

In some respects our present systems of units are very imperfect. Aside from the fact that the duodecimal system would have been much preferable to the decimal, a great opportunity was lost when the meter was taken as a decimal part of the earth's circumfer-

ence instead of being made equal to twice the distance passed over in the first second, by a body falling from rest, at sea level, at Paris, and in vacuo. This would have made the unit force the weight of unit mass, instead of $\frac{1}{981}$ of the weight of unit mass. We do not care whether a block of metal of one kilogramme mass has its side such and such a fraction of the earth's circumference or not, but it would be very convenient to know that 200 units of energy would give it a velocity of 20 units of length per second, and that unit work would be done in raising it through unit distance. True, the relation would not have held exactly over the whole earth, but neither is the cubic centimeter of water exactly equal in mass to the gramme (it being about two-tenths of one per cent. less than a gramme at ordinary temperatures), and the practical advantages of this latter approximate relation have not, to my knowledge, been disputed.

If the unit length had been taken as 981 of our present centimeters, the weight of unit mass would have been equal to the unit force to within two-tenths of one per cent. (*i. e.*, to within the same degree of accuracy with which a cubic centimeter of water at ordinary temperatures represents a gramme of mass), over practically all those portions of the earth where the mechanic arts have attained their highest development. At the equator this practical unit of work would be slightly deficient in size, a circumstance which some slight acquaintance with the inhabitants of tropical countries leads me to believe would not be highly resented by them.

In our electric and magnetic systems especially, the state of affairs is very deplorable. In addition to the two necessary systems of units, *i. e.*, the electrostatic and the electromagnetic, we have a third one, which is absolutely unnecessary, *i. e.*, the so-called practical system.

To express the relations between the units of this system and of the electromagnetic system a large number of constants of various sizes are employed. This creates great consequent confusion and annoyance.

Again, as Heavyside has pointed out, the definition of unit quantity of electricity and unit quantity of magnetism by the equations

$$\frac{QQ'}{\kappa L^2} = \text{Force}, \qquad \frac{PP'}{\mu L^2} = \text{Force},$$

instead of by the equations

$$\frac{QQ'}{4\pi\kappa L^2} = \text{Force}, \qquad \frac{PP'}{4\pi\mu L^2} = \text{Force},$$

has the effect of introducing the very objectionable constant 4π into many of our most frequently used and most important formulæ.

Another point, which is equally important, is that the use of the present defining equations, given above, necessitates the employment of units of electric and magnetic flux in addition to the units of quantity of electricity and quantity of magnetism, the units of flux being of the same nature as the units of quantity, but being only $\frac{1}{4}\pi$ times the size of the former. This again gives rise to two distinct sets of coefficients, differing from each other in the same way.

On examination it will be found that the number of totally unnecessary electric and magnetic quantities thus introduced is not less than 46 in number, and, moreover, we must have the same number of additional unnecessary symbols.

To take an example, If we wish to know what electrostatic flux corresponds to 3 coulombs, we must

1. Divide by 10 to reduce to absolute electromagnetic units.
2. Multiply by 3.10^{10} to reduce to electrostatic units.

3. Divide by 4π to reduce to units of flux, the result being,

$$3 \text{ coulombs} = 23873 + 10^4 \text{ electrostatic lines.}$$

It seems[1] that there is a very strong feeling amongst the electricians and electrical engineers of this and other countries in favor of a return to absolute units and the abandonment of the practical system. Although some of the practical units have been legalized there is nothing to prevent our giving names to the absolute units and using them exclusively for practical work.

There remains, however, the 4π difficulty. Heavyside has proposed to get rid of this constant, or rather, to transfer the "eruption of 4πs" to a place where it will do no harm, by making the unit of current $1/\sqrt{4\pi}$, and the unit of voltage $\sqrt{4\pi}$ times as large as the present units.

But whilst this would undoubtedly have been the best way, if it could have been done in 1873, it is now not practicable. It necessitates changing the ampère, coulomb, volt, ohm, farad and henry. Most of these have been legalized, and a very large sum has been locked up in standards. Were any attempt made to change the present legal units, the government would probably, and justly, point out that it is only a few years since the present units were recommended for adoption, and request its petitioners to excuse the taking of any further action until some evidence was furnished that the proposed alterations were likely to meet with a permanent approval.

I have found, however, that by a great piece of good fortune there exists a way in which the 4π can be got rid of without disturbing any of the legal units. It consists in shifting the 4π from the unit of quantity into the coefficient μ.

From the equation

$$\text{curl}\,(QP) = \text{Work}/T$$

we see that, there being but one curl, we have but one 4π. If we keep our units of mass, length and time unchanged, we can get rid of the 4π in three ways: 1. By dividing up the 4π between Q and P, *i. e.*, making each $1/\sqrt{4\pi}$ times as large as before. This was suggested by Heavyside. 2. By putting the 4π over on the other side, *i. e.*, making the unit of work 4π times as large as before. This is impracticable. 3. By making one of the two quantities, Q or P, $1/4\pi$ times as large as before.

The third method is the practical one. We cannot change the coulomb, but we can take the unit quantity of magnetism as equal to the present line, and the unit of difference of magnetic potential as equal to the current turn, without creating any appreciable disturbance with the existing order of things. Neither of these quantities has been defined by law, and as a matter of fact the ampère turn is already used by designers as a practical unit of gilbertance.

We now have

$$dQ/dT = \text{voltance}$$
$$dP/dT = \text{gilbertance}$$

and have, therefore, now no use for the "flux" as distinct from the "quantity." We have, therefore, no use for the quantities, specific inductive capacity and permeability, *i. e.*, for the ratios

$$\text{electric flux density} \div \text{volts per cm.}$$
$$\text{magnetic flux density} \div \text{gilberts per cm.}$$

whereas we still have use for the ratios

$$\text{quantity of electricity per sq. cm.} \div \text{volts per cm.}$$
$$\text{quantity of magnetism per sq. cm.} \div \text{gilberts per cm.}$$

[1] See in this connection the following articles: Editorial, Electrical World, June 9, 1899. Blondel, Electrical World, July 29, 1899, and the writer, Electromagnetic Design. Franklin Inst., April, 1899.

No names have been given to these quantities, though the ratio

intrinsic quantity of magnetism per sq. cm. ÷ gilberts per cm.

has been termed the "susceptibility."

We might use the present terms, specific inductive capacity and permeability, to denote these latter ratios. But it is always objectionable to give an old name to a new quantity, and I therefore suggest that the ratios

$$Q \text{ per sq. cm.} \div F, \text{ and } P \text{ per sq. cm.} \div H$$

be termed the "capity" and "permity" respectively. The similarity of the names will indicate their relationship to the quantities specific inductive capacity and permeability, and we may retain the symbols κ and μ for the quantities capity and permity. As mentioned above, we have introduced the 4π into the constant μ. The permity of a vacuum is then 4π, whereas the permeability for vacuo (or air. It should be vacuum, if v is given for vacuo and not for air) is now unity. The value of the electric constant remains the same as before, only, as we are now dealing with a different thing, *i. e.*, the ratio of quantity ÷ voltivity instead of flux ÷ voltivity, we now speak of the capity of a vacuum as being $\frac{1}{4}\pi$ where before we spoke of the specific inductive capacity of a vacuum as unity.

It will be seen that we have put the 4π where it will do no harm. The fact that we have now $\mu = 1846$ where before we had $\mu = 147$ is a matter of no consequence. No one uses published tables of permeability for designing, for no two samples of iron have the same value and as each sample comes into the factory it must have its constants determined just as if no other sample of iron had ever existed. As a matter of fact, most of our data with respect to slightly magnetic substances has been published in the shape of tables of susceptibility and not of permeability.

The same remarks apply to capacity. No one would think of using tables of capacities for making cables or condensers. Very little is known of the true capacity of substances, as this constant is affected to a very great extent by traces of moisture. I have myself been able to reduce Hopkinson's figures for oils very materially by using pure materials, carefully dried. As another example, the capacity of india rubber is given by Jenkin as 2.80, and by Salford and Halman as 3.7 whilst cable manufacturers, who use a special treatment to eliminate water, will furnish it as low as 2. No one would therefore think of using published tables for any practical or theoretical work of importance. In making cables or condensers a sample is always made up first and the capacity calculated from that.

I give below a table showing the relation between the present and suggested units and formulæ.

Present Units.	*Suggested Units.*
Coulomb	No change.
Ampère	"
Volt	"
Ohm	"
Farad	"
Erg. Joule, Watt	"
Specific Ind. Capacity.	No change, but we have now no use for the quantity specific inductive capacity, using capity instead.
Permeability.	$(4\pi)^2$ times as large. But having now no use for this quantity we use the permity, which is 4π for vacuo.
Quantity of Magnetism.	$\frac{1}{4}\pi$ times as large as before, *i. e.*, equals present line.
Difference of Magnetic Potent.	4π times as large as before.

Present Equations.	*Suggested Equations.*
$\frac{QQ'}{\kappa L^2} = \frac{PP'}{\mu L^2} =$ Force,	$\frac{QQ'}{4\pi\kappa L^2} = \frac{PP'}{4\pi\mu L^2} =$ Force.
$E = 4\pi\frac{dP}{dT}$, $G = 4\pi\frac{dQ}{dT}$,	$E = \frac{dP}{dT}$, $G = \frac{dQ}{dT}$,
$\frac{D^2}{8\pi\kappa} = \frac{B^2}{8\pi\mu} =$ Energy per c.c.	$\frac{D^2}{2\kappa} = \frac{B^2}{2\mu} =$ Energy per c.c.

I gave also a complete table of the quantities, symbols and units which would be necessary under the new system. It will be noted that 46 quantities, whose existence is now necessitated, owing to the present distinction between quantity and flux, have been dropped, as not required with the new system.

No importance is attached to the *names* suggested. They can, in most cases, be improved upon. Stress is however laid upon the symbols proposed, which it is thought will not readily be bettered. For vector quantities in the electric, magnetic and electromagnetic systems, block or script letters may be used, and ordinary type for scalers. Where, as in ælotropic substances, the capity, permity or conductivity has direction, cursives may be used. For the mechanical quantities, gravity and heat, italics are used. Vector quantities here may be either block or script. For radiation, German type is used.

The terminations have the following meanings.

a, Absolute
ma, Absolute electromagnetic.
sta, Absolute electrostatic.
l, amount per cm.
r, amount per sq. cm.
v, amount per cubic cm.
t, amount per second.
rt, amount per sec. per sec.
st, amount per sec. per sec. per sec.

The syllable *al* denotes extrinsic quantity.

The syllable *id*, denotes intrinsic quatity.

It will be noted that all the regular uncials are free for use for other purposes, as also the letters *d*, δ, Δ and *X*, *Y*, *Z*.

The idea of using terminations in this manner is of course not new. Quite recently, since the above was written, the matter has been brought forward again in a very interesting paper by M. Hospitalier.

The changes suggested can be made without interfering with any of the legal units, without rendering useless any tables used in practical work, and will merely render it necessary to alter the values of the electric constants of the few substances whose capacities we know accurately. In fact, even this is not necessary, for since we have made no change in the unit of capacity, but have merely introduced a new quantity, the capity as being more convenient, those who choose may still employ the present term, specific inductive capacity, and use the present values.

It may be mentioned that if, at some future time, we shall revise our units, taking the mass of a cubic cm. of the ether as the unit of mass, and twice the distance passed over in the first second by a body falling, from rest, in vacuo, at Paris, at sea level, as our new meter, we may use a system of symbols and names naturally provided for us.

For, it will be noted that all physical quantities have their qualities in the form

$$\frac{M^v L^x}{T^y L^z}$$

where x, y and z may have the value 1, 2 or 3, and v the value 0 or 1. Consequently all physical quantities can be symbolized by using the one single letter H with different lengths of arms.

Thus, F represents electric current per sq. cm.

I' represents energy per cubic cm.

H represents stress per unit area.

For writing, we may use / for 1, (for 2, and) for 3.

Different fonts of type can be used to distinguish between electric, magnetic and other quantities. At present this suggestion would hardly meet with approval from the printers

Similarly, if we take m for M, t for T, rt for T^2, st for T^3, a for L, e for L^2, i or y for L^3, u for two L's in the same direction and o or n for zero for each possible physical quantity we get a corresponding word.

Thus, Elmore would represent electric current.

Mury would represent energy per c. c.

Marte would represent stress per unit area.

The matter is, however, for the present at least, one of curiosity rather than of practical importance.

Addenda.

The coefficient η, referred to several times, is the coefficient in Steinmetz's formula

$$\text{Hysteresis loss per c. c. per cycle} = \eta B^{1.6}.$$

Having these relations between H, B, μ, η and the elasticity, we are in a position to develop the quantitative mathematical side much further. For example, we may substitute in a number of terms in the Lagrangian function treatment of J. J. Thomson (Dynam. App. to Phys. and Chem.), and obtain a much more complete idea of the relation between elastic strain and magnetization.

The qualitative formula for temperature, given above, *i. e.*, energy per atom, is for monatomic molecules. It is better given as energy per molecule.

The statement that μ varies with H whilst κ does not vary with F is not to be understood as meaning that change of F makes *no* change of κ, but that it is not of the nature required to make κ a compliancy. It will, however, be seen that κ should vary inversely as F^2, for $M/L^3 = T^2/L^2 \times ML/T^2$. This phenomenon has as yet not been discovered.

Electric.

Nature.	*Name.*	*Symbol.*	*Equation.*	*Suggested Name for Absolute Electro-Magnetic Unit.*	*Size of Suggested Absolute Unit.*
Quantity	Quantity	Q	$Q^2/4\pi\kappa L^2 = \Phi$	Coulma	Same as present
Quantity per sq. cm.	Surface Density	D	$D \times \text{area} = Q$	Coulmar	"
Quantity per c. c.	Volume Density	C	$C \times \text{volume} = Q$	Coulmav	"
Quantity Coefficient	Capity	κ	$\kappa F = D$		" (Note)
Recip. of do.	Recusity	λ	$F/\lambda = D$		"
Quantity Factor	Capance	K	$EK = Q$	Faradma	"
Recip. of do.	Recusance	Λ	$E/\Lambda = Q$	Darafma	"
Potential Difference	Voltance	E	$EQ = \Psi$	Volma	"
Potential Difference per cm.	Voltivity	F	$E \div \text{length} = F$	Volmal	"
Intrinsic Quantity	Quantidity	Q_i	$Q_i + Q_e = Q$	Coulidma	"
Extrinsic Quantity	Quantality	Q_e	$Q_e = EK_e$	Coulalma	"
Intrinsic Quantity Coefficient	Capidity	κ_i	$\kappa_i + \kappa_e = \kappa$		"
Recip. of do.	Recusidity	λ_i	$\lambda_i = 1/\kappa_i$		"
Intrinsic Quantity Factor	Capidance	K_i	$K_i + K_e = K$	Faridima	"
Recip. of do.	Recusidance	Λ_i	$\Lambda_i = 1/K_i$	Darafidma	"
Extrinsic Quantity Coefficient	Capality	κ_e	κ_e is κ of ether		"
Recip. of do.	Recusality	λ_e	λ_e is λ of ether		"
Extrinsic Quantity Factor	Capalance	K_e	K_e is K of ether	Faradalma	"
Recip. of do.	Recusalance	Λ_e	Λ_e is Λ of ether	Darafalma	"
Increase per degree C	Temperature Coefficient	κ_E, λ_E			

Note.—No importance is attached to the names suggested, these being given merely for the purpose of illustrating the system. The terminations -a, -ma and -sta stand for the terms "absolute unit," "absolute electro-magnetic unit" and "absolute electrostatic unit" respectively. The terminations -l, -r, -v, -t, -rt, -st stand for quantity per cm., per sq. cm., per c. c., per second, per sec. per sec. and per sec. per sec. per sec. respectively. The syllable "id" is used for "intrinsic quantity" and the syllable "al" for "extrinsic quantity."

MAGNETIC.

Name.	*Symbol.*	*Equation.*	*Suggested Name for Absolute Electro-Magnetic Unit.*	*Size of Suggested Absolute Unit.*
Quantity	P	$P^2/4\pi\mu L^2 = \phi$	Gausma	$1/4\pi$ times present *i. e.*, equal to present magnetic line
Surface Density	B	$B \times$ area $= P$	Gausmar	$1/4\pi$ times present
Volume Density	A	$A \times$ volume $= P$	Gausmav	$1/4\pi$ times present
Permity	μ	$\mu H = B$		$(4\pi)^2$ times present
Reluctity	ν	$H/\nu = B$		$1/(4\pi)^2$ " "
Permance	M	$GM = P$	Rema	$(4\pi)^2$ " " (Note 2)
Reluctance	N	$G/N = P$	Oersma	$1/(4\pi)^2$ " "
Gilbertance	G	$GP = \psi$	Gilma	4π times present *i. e.*, equal to amma turn
Gilbertivity	H	G ÷ length $= H$	Gilmal	4π times present
Quantidity	P_i	$P_i + P_e = P$	Gausioma	$1/4\pi$ times present
Quantality	P_e	$P_e = GM_e$	Gausalma	$1/4\pi$ times present
Permidity	μ_i	$\mu_i + \mu_e = \mu$		$(4\pi)^2$ times present
Reluctidity	ν_i	$\nu_i = 1/\mu_i$		$1/(4\pi)^2$ " "
Permidance	M_i	$M_i + M_e = M$	Remida	$(4\pi)^2$ " "
Reluctidance	N_i	$N_i = 1/M_i$	Oersidma	$1/(4\pi)^2$ " "
Permality	μ_e	μ_e is μ of ether		$(4\pi)^2$ " "
Reluctality	ν_e	ν_e is ν of ether		$1/(4\pi)^2$ " "
Permalance	M_e	M_e is M of ether	Remalma	$(4\pi)^2$ " "
Reluctance	N_e	N_e'' is N of ether	Oersalma	$1/(4\pi)^2$ " "
Hysteresis Coefficient	η		Hergav.	
Temperature Coefficient	μ_E, ν_E			

2. In our present system, the permity of air is $1/4\pi$.

Electro-Magnetic.

Nature.	*Name.*	*Symbol.*	*Equation.*	*Suggested Name for Absolute Electro-Mag. Unit.*	*Size of Suggested Absolute Unit.*
Quantity of Electricity per sec.	Current	I	$I = dQ/dT$	Amperma	Same as present.
Q'ntity of Electricity per sq. cm. per sec.	Current Density	J	$J = I \div$ area	Ampermar	"
Quantity of Magnetism per sec.	Flow	U	$U = dP/dT$	Weberma	$1/4\pi$ times present size *i. e.*, equal to present line per sec.
Q'ntity of Magnetism per sq. cm. per sec.	Flow Density	V	$V = U \div$ area	Webermar	$1/4\pi$ times present size.
Conductivity	Conductivity	σ	$F\sigma = J$		No change.
Recip. of do.	Resistivity	ρ	$F/\rho = J$		"
Conductance	Conductance	S	$ES = I$	Moma	"
Recip. of do.	Resistance	R	$E/R = I$	Ohma	"
Quantity of Electricity $\times$ turns					"
Quantity of Magnetism $\times$ turns	Linkage	T	$T = P \times$ turns	Maxawelma	"
Linkage $\div$ current	Inductance	L	$L = T \div I$	Henryma	"
Imaginary Resistance due to Capance	Electric Reactance	R_K	$R_K = 1 \div K\omega$	Ohmak	"
Recip. of do.	Electric Permittance	S_K	$S_K = K\omega$	Momak	"
Imaginary Resistance due to Inductance	Magnetic Reactance	R_M	$R_M = L\omega$	Ohmam	"
Recip. of do.	Magnetic Permittance	S_M	$S_M = 1 \div L\omega$	Momam	"
Total Imaginary Resistance	Reactance	R_j	$R_j = L\omega + 1/K\omega$	Ohmaj	"
Recip. of do.	Permittance	S_j	$S_j = K\omega/(KL\omega^2 + 1)$	Momaj	"
Apparent Resistance	Impedance	R_v	$R_v = \sqrt{R^2 + (L\omega - 1/K\omega)^2}$	Ohman	"
Recip. of do.	Admittance	S_v	$S_v = 1/\sqrt{R^2 + (L\omega + 1/K\omega)^2}$	Moman	"
Increase per degree C.	Temperature Coeff.	σ_E, ρ_E			

HEAT.

Nature.	*Name.*	*Symbol.*	*Equation.*	*Unit.*
Quantity	Heat	Q	$Q = KE$	Erg.
Quantity per cc.	Heat Density	C	$C = Q \div V$	Ergav
Quantity Coefficient	Specific Heat	k	$kEV = Q$	
Quantity Factor	Heat Capacitance	K	$k \times V = Q$	
Potential Difference	Temperature Difference	E	$Ek = Q$	Degree
Potential Difference per cm.	Temperature Gradient	F	$E \div L = F$	Degreel
Quantity per sec.	Flux	I	$I = Q \mid T$	Ergat
Quantity per sq. cm.	Flux Density	J	$J = I \div A$	Ergart
Flux Coefficient	Impartivity	s'	$Fs = J$	
Recip of do.	Morativity	r	$F \mid r = J$	
Flux Factor	Impartance	S	$ES = I$	Foura
Recip of do.	Moratance	R	$E \mid R = I$	Ruofa
Diffusion Coefficient	Diffusivity	s_E	$s_E = s \div k$	
Quantity Coefficient Constant Volume	Specific Heat, Constant Volume	k_v		
Quantity Coefficient Constant Pressure	Specific Heat, Constant Pressure	k_p		
Critical Constants	Critical Temperature	E_c		
	Critical Pressure	Π_c		
	Critical Volume	V_c		
Coefficient of	Linear Expansion	L_E		
" "	Surface "	A_E		
" "	Volume "	V_E		

Radiation.

Nature.	*Name.*	*Symbol.*	*Equation.*	*Unit.*
Quantity of Radiation Emitted	Radiance	$\mathfrak{Q}$	$\mathfrak{g}\mathfrak{Q} = \mathfrak{P}$	Erg.
Quantity per sq. cm.	Radivity	$\mathfrak{D}$	$\mathfrak{D} \times \text{area} = \mathfrak{Q}$	Ergar
Quantity of Light Emitted	Lumenescence	$\mathfrak{P}$	$\mathfrak{P} = \mathfrak{g}\mathfrak{Q}$	
Quantity per sq. cm.	Lumenosity	$\mathfrak{B}$	$\mathfrak{B} \times \text{area} = \mathfrak{P}$	
Radiance per Degree C.	Emittance	$\mathfrak{E}$	$\mathfrak{E}\mathfrak{E} = \mathfrak{Q}$	
Radiance per sq. cm.	Emissivity	$\mathfrak{h}$	$\mathfrak{h}\mathfrak{E} = \mathfrak{D}$	
Lumenance per Degree C.	Lumenance	$\mathfrak{M}$	$\mathfrak{M}\mathfrak{E} = \mathfrak{P}$	
Lumenance per sq. cm. per Degree C.	Lumenity	$\mathfrak{m}$	$\mathfrak{m}\mathfrak{E} = \mathfrak{B}$	
Radiance Received	Receptance	$\mathfrak{q}$	$\mathfrak{r}\mathfrak{q} = \mathfrak{J}$	
Radiance per sq. cm.	Receptivity	$\mathfrak{d}$	$\mathfrak{d} \times \text{area} = \mathfrak{q}$	
Lumenescence Received	Luxance	$\mathfrak{p}$	$\mathfrak{f}\mathfrak{p} = \mathfrak{A}$	
Lumenescence per sq. cm.	Luxity	$\mathfrak{b}$	$\mathfrak{b} \times \text{area} = \mathfrak{p}$	
Radiance Absorbed	Absorbance	$\mathfrak{J}$	$\mathfrak{J} = \mathfrak{e}\mathfrak{q}$	
Radiance per sq. cm.	Absorbity	$\mathfrak{J}$	$\mathfrak{J} \times \text{area} = \mathfrak{J}$	
Lumenescence Absorbed	Illumance	$\mathfrak{A}$	$\mathfrak{A} = \mathfrak{f}\mathfrak{p}$	
Lumenescence per sq. cm.	Illumity	$\mathfrak{V}$	$\mathfrak{V} \times \text{area} = \mathfrak{A}$	
Absorbity ÷ Receptivity	Absorbance Efficiency	$\mathfrak{e}$	$\mathfrak{e} = \mathfrak{J} \div \mathfrak{d}$	
Illumity ÷ Luxity	Luxance Efficiency	$\mathfrak{f}$	$\mathfrak{f} = \mathfrak{V} \div \mathfrak{b}$	
Lumenity ÷ Radivity	Luminous Efficiency	$\mathfrak{g}$	$\mathfrak{g} = \mathfrak{B} \div \mathfrak{P}$	

GRAVITY.

Nature.	*Symbol.*	*Nature.*	*Symbol.*
Gravity	G	Acceleration of Gravity	i
Quantity Coefficient	γ	Ponderate Acceleration (note)	g

MECHANICS.

Nature.	*Symbol*	*Nature.*	*Symbol.*
Mass	M	Angular Velocity	ω
Length	L	Work	Ψ
Area	R	Work per cm. or Force	Φ
Volume	V	Work per sq. cm. Surface Tension	Σ
Density	ς	Work per c. c. or Pressure	Ξ
Velocity	v	Work per second	Π
Periodicity	τ	Force per cm.	Γ
Acceleration	a	Young's Modulus	ζ
Plane Angle	φ, θ	Rigidity	ξ
Solid Angle	Ω	Tensile strength	χ

NOTE.—*I. e.*, acceleration due to gravity plus that due to rotation of earth on its axis, around the sun, etc.

THERMODYNAMIC NOTES (NO. 10): REVISION OF SOME THERMAL DATA CONCERNING BENZENE.[1]

K. TSURUTA.

PROFESSOR S. YOUNG[2] has given full measurements on vapor pressure (p) and specific volumes (v_l liquid and v_g vapor) of benzene in its saturated states. The values of $\frac{dp}{dT}$ were found from Biot's formula used by him, and the heat of vaporization (r or r_{lg}) was calculated, the results being given in the following table, in which for reference, some of the original data are reproduced.[3]

HEAT OF VAPORIZATION r (ACCORDING TO PROFESSOR S. YOUNG).

t	v_l	v_g/v_l	p	r	t	v_l	v_g/v_l	p	r
	c.c.		*m.m.*	*Cal.*		*c.c.*		*m.m.*	*Cal.*
80°C.	1.2278	298.0	753.62	95.34	180°C.	1.4480	27.90	7625.2	74.72
90	1.2436	225.0	1016.1	94.45	190	1.4798	22.70	9049.4	71.89
100	1.2615	169.0	1344.3	91.60	200	1.5139	18.65	10663.0	69.01
110	1.2806	129.0	1748.2	89.02	210	1.5546	15.23	12482.0	65.75
120	1.3000	100.0	2238.1	86.41	220	1.5986	12.47	14526.0	62.23
130	1.3214	79.4	2824.9	84.78	230	1.6487	10.15	16815.0	58.44
140	1.3440	{ 62.7 63.60	3520.0	{ 82.06 83.25	240	1.7091	8.21	19369.0	54.13
150	1.3680	{ 50.5 50.90	4334.8	{ 80.17 80.82	250	1.7827	6.55	22214.0	49.36
160	1.3918	42.10	5281.9	80.31	260	1.8770	5.14	25376.0	43.79
170	1.4198	34.30	6374.1	78.13	270	2.0063	3.88	28885.0	36.90

We reproduce again the direct measurements of r by Griffiths and Marshall :[4]

[1] It may be remarked that this revision was prepared some years ago for some cryoscopic experimental investigations with benzene which I had intended to carry on.

[2] S. Young, *Jour. of Chem. Society*, Vol. LV., (1899) and Vol. LIX. (1891).

[3] The unit quantity of heat here adopted is about 4.189×10^6 ergs (A. Griffiths, *Phil. Mag.*, Vol. XL. (1858), p. 453).

[4] A. Griffiths and Marshall, *Phil. Mag.*, Vol. XLI. (1896).

HEAT OF VAPORIZATION r (ACCORDING TO GRIFFITHS AND MARSHALL).

t (Nitrogen Scale).	r Cal.	t (Nitrogen Scale).	r Cal.
19°.947C.	103.82	40°.045C.	100.71±0.057
30 .066	102.30±0.076	50 .014	99.14±0.034

The authors give the following linear formula as showing a very close approximation to the experimental values:

$$r = 107.05 - 0.1581 \times t.$$

When we plot these values (indicated by "experi." in the following table), it will be found that between 180° C. and 270° C. the representative points are regularly arranged, but quite irregularly between 80° C. and 180° C., showing, perhaps, that with Professor Young's sealed tube method "the results are more reliable at high than at low temperature." (Professor Young). Connecting them, however, by a mean curve I obtained the numbers (indicating by "curve") in the following table:

HEAT OF VAPORIZATION r (ACCORDING TO WRITER'S MEAN CURVE).

t	r (Curve.) Cal.	r (Experi.) Cal.	t	r (Curve.) Cal.	r (Experi.) Cal.	t	r (Curve.) Cal.	r (Experi.) Cal.
10°	105.24	———	100°	91.05	91.60	190°	71.93	71.89
20	103.75	103.82	110	89.20	89.02	200	69.01	69.01
30	102.26	102.30	120	87.36	86.41	210	65.80	65.75
40	100.76	100.71	130	85.43	84.78	220	62.32	62.23
50	99.24	99.14	140	83.48	82.06 83.25	230	58.49	58.44
60	97.65	———	150	81.35	80.17 80.82	240	54.21	54.13
70	96.04	———	160	79.20	80.31	250	49.40	49.36
80	94.40	95.34	170	76.90	78.13	260	43.76	43.79
90	92.76	94.45	180	74.53	74.72	270	36.97	36.90

It will be seen from this table that my mean curve can be used for the purpose of interpolation and hence for some other purposes.

From these values of r we can calculate backwards the ratio of $\frac{v_i}{v_g}$ and then v_- taking as correct Professor Young's experimental

values of v_1. I did this and found that the "mean values" of densities of the liquid and vapor obey in quite a satisfactory manner the empirical "law of straight diameter" (Cailletet and Mathias), not only from 80° C. to the critical temperature as remarked by Professor Young himself, but also from 80° C. down to the triple point. A more minute examination showed, however, the so-called diameter, in the case of *my* mean curve, to consist of two nearly straight parts meeting somewhere near the temperature 180° C., or more strictly, to be a curve whose curvature is so slight that it can be so described. This may indicate that *my* mean curve is not a good mean curve. But since we should ascribe much importance to the experimental values of Griffiths and Marshall, and surely to those that have been deduced from Professor Young's measurements between 180° C. and 270° C., I think that we must rather regard the empirical "law of straight diameter" as holding good in first approximation, but not in so reliable a degree as to draw from it any thermodynamic conclusions concerning benzene.

Another important source of information is the formula given by Regnault for the total heat (λ):

$$\lambda = 109.0 + 0.24429 \times t - 0.0001315 \times t^2.$$

To make it comparable with the above results of calculation, and then to make some applications of it, it must first of all be subject to some thermometric and calorimetric reductions. Then by interpolations it would become possible to calculate, if we make use of the above result, the specific heat of saturated vapor, and to obtain perhaps quite a satisfactory formula for the specific heat of liquid benzene at higher temperatures; or to calculate, if we take the formula in conjunction with some reliable measurements on the specific heat of liquid benzene, the heat of vaporization at various temperatures, and then see how far do the results coincide with the calculations of this kind given by Wüllner.[1] As, however, I do not possess Regnault's original memoirs, I was unable to examine these points.

[1] A. Wüllner, Experimental Physik, Bd. III. (4to Auf.), 5, 726.

Let us now turn to the vapor pressures of benzene below its triple point (+ 5°.3 C.).

The measurements of these by Fischer,[1] Ramsay and Young,[2] and Ferche[3] were plotted on section paper and mean curves drawn to pass well through the representative points. This was done twice quite independently, and when the results obtained were compared, it was found that for liquid benzene they differed very little in the cases of Fischer's and Ferche's measurements, generally by 0.01–0.03 mm. of mercury; but with Professor Ramsay and Young's they differed sometimes by 1.5 mm., perhaps owing to a comparatively small number of points available for drawing the mean curve between the triple point and −1° C.

VAPOR PRESSURE OF LIQUID BENZENE.

t	Fischer.	R. & Y.	Ferche.	Mean Curve.	*t*	Fischer.	R. & Y.	Ferche.	Mean Curve.
	m.m.	*m.m.*	*m.m.*	*m.m.*		*m.m.*	*m.m.*	*m.m.*	*m.m.*
− ° 1	24.87	25.04	25.03	24.98	+2°.5	30.28	30.25	30.37	30.30
−0 .6	25.60	25.73	25.68	25.67	3	31.11	31.11	31.23	31.55
0	26.34	26.42	26.41	26.39	3	31.99	32.00	32.10	32.03
+0 .5	27.10	27.14	27.16	27.13	3 .5	32.87	32.94	33.01	32.94
1	27.88	27.88	27.91	27.89	4	33.78	33.89	33.94	33.87
1 .5	28.66	28.65	28.69	28.67	4 .5	34.70	34.88	34.87	34.82
2	29.46	29.43	29.52	29.47	5				

Proceeding in a similar way with solid benzene, I obtained the following values from the different authors' sets of measurements. Things are less favorable in the present case than in the preceding, in fact, not only did my own mean curves, several of which I drew for one set, differ much among themselves in the cases of Professor Ramsey and Young's and Fischer's measurements, but also the different sets compared with one another showed greater discrepancies than in the preceding case. It will be seen that my mean curve runs on the whole quite close to Ferche's, that of Fischer lying nearly as much above the latter as Professor Ramsey and Young's lies below it.

[1] W. Fischer, *Wiede. Ann.*, Bd. 28 (1886).
[2] W. Ramsay and S. Young, *Phil. Mag.*, Vol. 23 (1887).
[3] J. Ferche, *Wiede. Ann.*, Bd. 44 (1891).

VAPOR PRESSURE OF SOLID BENZINE.

t	Fischer.	R. & Y.	Ferche.	Mean Curve.	t	Fischer.	R. & Y.	Ferche.	Mean Curve.
	m.m.	m.m.	m.m.	m.m.		m.m.	m.m.	m.m.	m.m.
−1°C.	23.21	21.97	22.52	22.57	+2.5C	29.30	28.97	29.14	29.14
−0.5	24.03	22.94	23.40	23.46	3	30.25	30.05	30.19	30.16
0	24.88	23.90	24.30	24.36	3.5	31.25	31.16	31.27	31.23
+0.5	25.72	24.89	25.23	25.28	4	32.28	32.32	32.36	32.32
1	26.58	25.88	26.18	26.21	4.5	33.35	33.52	33.50	33.46
1.5	27.46	26.89	27.13	27.16	5	34.50	34.75	34.68	34.64
2	28.36	27.91	28.12	28.13					

Following the line of reasoning used by Guldberg[1] we can obtain a thermodynamic relation between p_i and p_s. By the fundamental principles of reversible thermodynamics considered as applicable to a substance in any of the two coexisting phases of it, we have the well known formulæ: in the case of liquid and vapor phases.

$$r_{gi} = T(v_g - v_i)\cdot\frac{dp_i}{dT},$$

and

$$h_{gi} = T.d\frac{\left(\frac{r_{gi}}{T}\right)}{dT} + c_i;$$

in the case of vapor and solid phases,

$$r_{gs} = T(v_g - v_s)\cdot\frac{dp_s}{dT},$$

and

$$h_{gs} = T.d\frac{\left(\frac{r_{gs}}{T}\right)}{dT} + c_s,$$

where $h_{gl}(h_{gs})$ and $c_l(c_s)$ develop respectively the specific heats along vapor- and liquid- (solid-) lines. Assuming the vapor over the liquid and solid states to be the same, we may put

$$h_{gi} = h_{gs},$$

so that

$$c_i - c_s + T\frac{d}{dT}\left(\frac{r_{gi} - r_{gs}}{T}\right) = 0.$$

By integration we obtain

$$\int\frac{c_i - c_s}{T}dT = \frac{r_{gi} - r_{gs}}{T} + k \text{ (const.)}.$$

[1] Guldberg, C. R. (1870).

Assuming $c_i - c_s$ as constant we obtain

$$(c_i - c_s) \log T = \frac{r_{gs} - r_{gi}}{T} + k,$$

which gives $$(c_i - c_s) \log \frac{T_0}{T} = \frac{F_0}{T_0} - \frac{1}{T} r_{gs} - r_{gi}),$$

where $F_0 (= r_{gs} - r_{gi}$ exactly at the triple point) denotes the heat of fusion at that temperature. When we neglect v_i as well as v_s in comparison with v_g and make the further assumption that Boyle-Gay-Lussac's law together with Avogadro's "hypothesis" holds good for the vapor of benzene between the triple point and near o°C., we obtain

$$\frac{1.971}{78} \cdot T \cdot \frac{d}{dT} \left(\log \frac{p_s}{p_i} \right) = \frac{F_0}{T_0} - (c_i - c_s) \log \frac{T_0}{T} .$$

Hence, by integration and some transformation,

$$\log \frac{p_s}{p_i} = \frac{78}{1,971} \cdot \log \frac{T}{T_0} \cdot \left(\frac{F_0}{T_0} + \frac{c_i - c_s}{2} \log \frac{T}{T_0} \right),$$

which becomes, when we make some neglections,

$$\log \frac{p_s}{p_i} = \frac{78}{1,971} \cdot \frac{\tau \cdot F_0}{T_0^2} ,$$

exactly as given by Guldberg.

In the last but one formula I have put

$$\left.\begin{aligned} T_0 &= 278.3, \\ F_0 &= 30.00, \\ c_i &= 0.3350, \\ c_s &= 0.2032 \end{aligned}\right\} \text{(Ferche.)}^1$$

and obtained the numbers given in the first three columns of the following table:

[1] Here we may put c_i and c_s equal to the specific heats at constant pressure of liquid and solid benzene. M. Fischer puts the specific heat of solid benzene equal to 0.319, while M. Ponsot (C. R., 1894) adopts the value 0.017 of the difference of the two specific heats without giving any reason for it. Though of no serious effect in the present case, yet such a difference considered in itself requires an experimental examination.

VAPOR PRESSURES OF SOLID AND LIQUID BENZENE.

t	p_s	$\frac{p_i}{p_s}$	p_s (Fischer.) minus.	p_s (R. & Y.) minus.	p_s (Ferche) minus.	p_s (Mean.) minus.
	Calculated.	Calculated.	p_s (Calc.)	p_s (Calc.)	p_s (Calc).	p_s (Calc.)
	m.m.		m.m.	m.m.	m.m.	m.m.
−1° C.	22.69	1.1011	+0.52	−0.72	−0.17	−0.12
−0 .5	23.49	1.0927	+0.54	−0.55	−0.09	−0.03
0	24.34	1.0844	+0.53	−0.44	−0.04	+0.02
+0 .5	25.21	1.0761	+0.51	−0.32	+0.02	+0.07
1	26.11	1.0680	+0.47	−0.23	+0.07	+0.10
1 .5	27.05	1.0599	+0.41	−0.16	+0.08	+0.11
2	28.01	1.0518	+0.35	−0.10	+0.11	+0.12
2 .5	29.03	1.0438	+0.27	−0.06	+0.11	+0.11
3	30.07	1.0359	+0.18	−0.02	+0.12	+0.09
3 .5	31.16	1.0280	+0.09	0	+0.11	+0.07
4	32.29	1.0201	−0.01	+0.03	+0.07	+0.03
4 .5	33.46	1.0123	−0.01	+0.06	+0.04	0
5	34.66	1.0046	−0.16	+0.09	+0.02	−0.02

The numbers in the last four colums may be taken as indicating teat the several assumptions we have made in deducing the formula employed as well as the several experimental data we have used in calculating the vapor pressures, are both of them not very wrong. Hence the formula with the data may be used as one of first approximation.[1]

TOKYO, JAPAN.

[1] By the way, let me here add the following. I obtained the following values of the constants of Biot's form of interpolation formula; choosing the mean values of pressure at $-0°.5$, $+2°$, and $+4°.5$ C. for liquid benzene:

$$\log p_i = a + ba^t,$$
$$a = \overline{7}.\ 8749340,$$
$$\log\ b = 0.\ 8770540,$$
$$\log\ a = 0.\ 0013878.$$

Similarly for solid benzenes:

$$\log p_s = a' + b'\beta^t$$
$$a' = 3.\ 153721,$$
$$\log b' = 0.\ 2512469,$$
$$\log\ \beta = \overline{1}.\ 9921452.$$

AN EFFICIENT BOYLE'S-LAW APPARATUS.

W. J. Humphreys.

NOTHING is more discouraging to teacher or student than a piece of apparatus incapable of giving consistent and approximately accurate results. For this reason the ordinary U shaped tube, used for demonstrating the truth of Boyle's law, is one of the least satisfactory things the junior laboratory student has to work with. No matter how careful his measurements, irregularities in the bore of the tube, even where all other sources of appreciable error are avoided, will lead to irregular and unsatisfactory determinations.

The importance of apparatus capable of yielding good results, however elementary the experiments, is my excuse for offering the following description of a Boyle's law device which I have recently had constructed, and which, according to tests, readily gives values that do not differ from the correct ones by more than one part in a thousand.

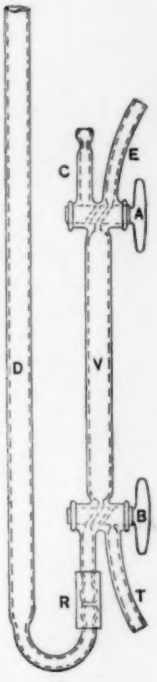


Fig. 1.

Referring to the figure : *V* is a tube about twelve centimeters long and one centimeter in diameter having a two-way stop-cock at eachend. *D* is a long tube of about the same diameter as *V* to avoid any appreciable difference in capillary action. *D* may be sealed to *V* or connected to it in any way desired—the figure shows the method I have adopted. In the event of any leakage the short rubber tube *R* may be made tight either by wrapping or by putting it on the glass tubes with shellac varnish. However I have not found either of these precautions necessary.

The stop-cocks, of course, must be air tight. An excellent lubri-

cant for them is paraffine, with just enough vaseline added to keep it from being brittle.

The operation is as follows: Close stop-cock B and fill D to any desired height with mercury, then open B and allow a small amount of mercury to run into V, then turn B till V and T are in communication. This lets the mercury out of V but leaves the tube through B that connects V and D entirely filled. Now close A and the volume of air in V is perfectly definite and at atmospheric pressure. Next turn B till V and D are again in communication. The mercury will now rise slowly in V and be free from all air bubbles. When the mercury has come to rest close B and read the difference in mercury levels in D and V, either with a cathetometer or with a meter bar placed between D and V. Now open A and B and run the mercury out T and weigh it. Obtain similar results for as many different heights as desired.

Lastly leave V open to E say and run mercury up V and into A, close B, open V to C, then run the mercury out T and weigh. This gives the weight of the mercury necessary to fill V absolutely.

Let W = weight of mercury required to fill V,
" w = " " " run in when A is closed,
" p = difference in heights of mercury in D and V,
" B = height of barometer at time of experiment,
" V = volume of tube V,
" v = " " compressed gas in V.

Then
$$\frac{W}{W-w} = \frac{V}{v} = \frac{B+p}{B},$$

and
$$B = \frac{p(W-w)}{w}.$$

Of course B may be obtained directly from a barometer, or it can be determined to within a fraction of a millimeter by the method just shown, its value substituted in the equations, and Boyle's law tested.

This is one method, and occasion to use it might arise, of obtaining a fairly satisfactory barometric reading without the use of a standard barometer. I have described the apparatus as used for pressures above one atmosphere. It can be used, however, with

equal ease and accuracy for pressures less than atmospheric, but the operations are so obvious and simple that a description of them would be superfluous, except possibly the statement that it is necessary to determine the weight of the mercury required to fill the tube in B that connects V with T.

If the student has time he may simply assume that, at constant temperature, the volume of a gas is some fixed function of the pressure. His values will show him what this function is, determine the height of the barometer, and in every way give him a most satisfactory demonstration of the truth of Boyle's law.

It may be not out of place to say that in filling D it is advisable to introduce the mercury through a small funnel or thistle tube, with its stem drawn out to a fine opening. This prevents fouling the walls of the tube, the dirt being collected on the sides of the funnel.

ROUSS PHYSICAL LABORATORY, UNIVERSITY OF VIRGINIA, Dec. 7, 1899.

NEW BOOKS.

Strahlung und Temperatur der Sonne. By DR. J. SCHEINER. Pp. i–v, 1–99. W. Engelmann, Leipzig, 1899.

This monograph on solar temperatures shows the importance which is now attached to Stefan's law, particularly in Germany. Some time has elapsed since Stefan noticed that the observations of Dulong and Petit, de la Provostage and Desains, Draper, Ericsson and others could be satisfactorily interpreted by assuming the quantity of heat radiated by a body to be proportional to the fourth power of its absolute temperature. Boltzmann thereafter deduced the same law from the electromagnetic theory of light by aid of the second law of thermodynamics. Boltzmann himself insisted on the provisional character of his proof; but the law has recently received independent experimental confirmation at the hands of Lummer and Pringsheim (1560° C.) and others, operating with absolutely black bodies, *i. e.*, with hollow equally heated radiators emitting heat from within. For such bodies only the law appears to be valid.

Dr. Scheiner, therefore, assumes the truth of Stefan's law to this extent, and defines the sun's effective temperature, as the temperature which it would have if it radiated like a black body. From the known value of the solar constant, s (heat received per sq. cm., per sec.) and the known heat emission, h, of a black body for a definite temperature interval (100°, Kurlbaum and others), it is thus possible to arrive at trustworthy values for the temperature of the sun t. If φ be the apparent solar diameter, Stefan's law leads approximately to

$$t = 273 \sqrt[4]{\frac{2.843}{\sin^2 \frac{\varphi}{2}} \frac{s}{h}},$$

and Scheiner in this way finds $t = 7010°$. Treating all earlier observations by the same method he computes the following list of independent values:

Pouillet,	$t = 5600°$	Langley,	$t = 6000°$
Secchi,	5400°	Wilson & Gray,	6200°
Violle,	6200°	etc.	
Soret,	5500°		

In view of the enormous discrepancies in the values deduced for the sun's temperature by these observers themselves (data between 1500° and

10,000,000°), the author concludes that the consistent values of the present method must commend the problem to the serious consideration of astronomers. The reviewer fails to see however, why similar data similarly interpreted, do not necessarily lead to comparable results.

Corrected for solar absorption, Scheiner's values for the effective temperature of the sun's photosphere is 7760° C.

Among many indirect methods for estimating the sun's temperature, the one based on the position (λ) of the energy maximum in the solar spectrum deserves mention here. Langley's observations (maximum at $\lambda = 0{\cdot}6\mu$) interpreted by Wien put the sun's effective temperature at 4630°, in reasonable agreement with the above data.

The monograph contains a full digest of correlative data in solar physics. The absorption of the earth's atmosphere is treated in detail. A complete account is given of solar radiation, considered both photometrically and thermally, including a summary of the laws of radiation hitherto proposed. Much space is devoted to the distribution of light over the sun's disc, to the absorption of the solar atmosphere, to secular and periodic changes of the sun's temperature, to the electrodynamic radiation of the sun, etc. Any student interested in the profound cosmical problems treated will profit by the varied information given in this useful little book.

CARL BARUS.

Geschichte der Physikalischen Experimentierkunst. By E. GERLAND UND F. TRAUMÜLLER. Pp. 442. (14 M.; bound, 17 M.). W. Engelmann, Leipzig, 1899.

The general history of physics embraces not only physical theories, but also the discovery of physical facts upon which the theories rest and the art of experimentation, by which the facts were ascertained. The above book limits itself to the last of these functions. The authors trace the history of the art of experimentation from early times down to the beginning of the present century. The 425 figures show some ingenious devices which are not usually explained in historical works. Here we find illustrated the ancient Egyptian process of glass-blowing, the Archimedian endless screw, several of Hero's curious devices, the famous "tower of the winds" in Athens, which testifies to an early interest in meteorology, the Roman steelyards, the hydrometer of Synesius, Al Khâzinî's "balance of wisdom," Gilbert's "instrumentum declinationis," Galileo's pendulum clock, Guericke's quaint illustrations of experiments on atmospheric pressure, Agricola's and Papin's mine ventilators, and many kinds of apparatus devised during more recent times. The work of the nineteenth century remains untouched, except the researches on electricity and magnetism during the first forty years.

The experiments of the *Accademia del Cimento* of Florence are described with greater fullness than we have seen in other histories. Extremely interesting are the series of experiments on the compressibility of water, culminating in the well-known test with the hollow spheres of silver which were filled with water, closed tight, and then disfigured by hammering until the water forced its way through the pores (or cracks?) of the metal. That the Academicians did not feel justified by their experiments to draw the conclusion that water is incompressible, speaks highly for the caution which they exercised in their researches. Their report on the subject ends as follows: "It is certain that, as compared with air, water withstands an infinitely greater pressure (if we may so express ourselves), which confirms what we said at the beginning. While experimental research does not always lead us to the most remote truths which we seek, it gives us, nevertheless, in each favorable or unfavorable case, some light, be it ever so little."

The struggle for supremacy between alcohol and mercury as thermometric substances is told in an attractive manner, but the authors fail to record the interesting fact brought to light by M. Maze (*Comptes Rendus*, Vol. 120, 1895, p. 732), that in 1559 more than half a century before Fahrenheit, a mercury thermometer was constructed by the astronomer Boullian.

The learned German authors point to Seebeck as one who approached closely to the discovery of Ohm's Law three years before Ohm made his experiments in 1826, but they have overlooked the fact established by Maxwell (*Electrical Researches of the Hon. Henry Cavendish*, 1879, p. LIX.), that Cavendish as early as 1781 completed a series of experiments which amount to an anticipation of Ohm's Law.

The authors describe the electrical researches of Faraday, but do not mention Joseph Henry, who, while a teacher at the Albany Academy, made the discovery of magneto-electricity about the same time as Faraday, and who antedated Faraday in the discovery of self-induction. The authors also speak of Morse without mentioning Henry, although it is quite firmly established that Morse's assistant, Dr. Gale, applied the principle discovered by Henry to render Morse's machine effective at a distance. (*Smithsonian Report* 1857, pp. 99–106.)

But these are minor blemishes. The work as a whole is accurate and up-to-date in the information which it imparts. It contains much material otherwise difficult of access.

Florian Cajori.

Colorado College, Colorado Springs.